Lecture 15

Dijkstra's Algorithm (contd.)

Source: CLRS

Dijkstra's Algorithm: Sketch

Computing $\pi[v] = \min_{(u,v)\in E,\ u\in S} d[u] + w(u, v)$

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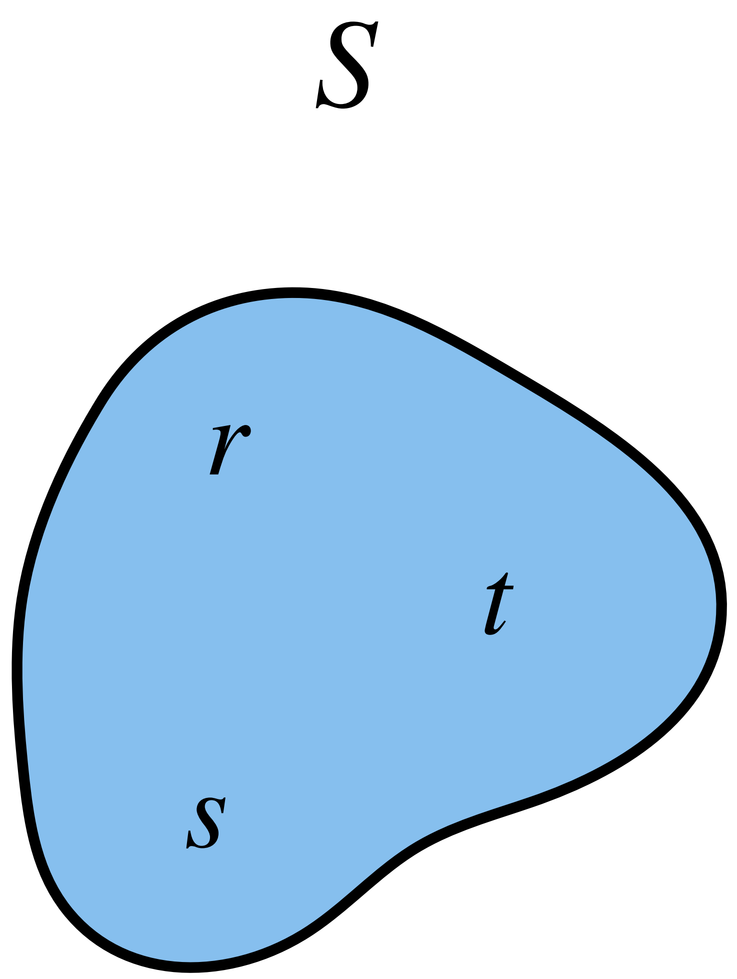

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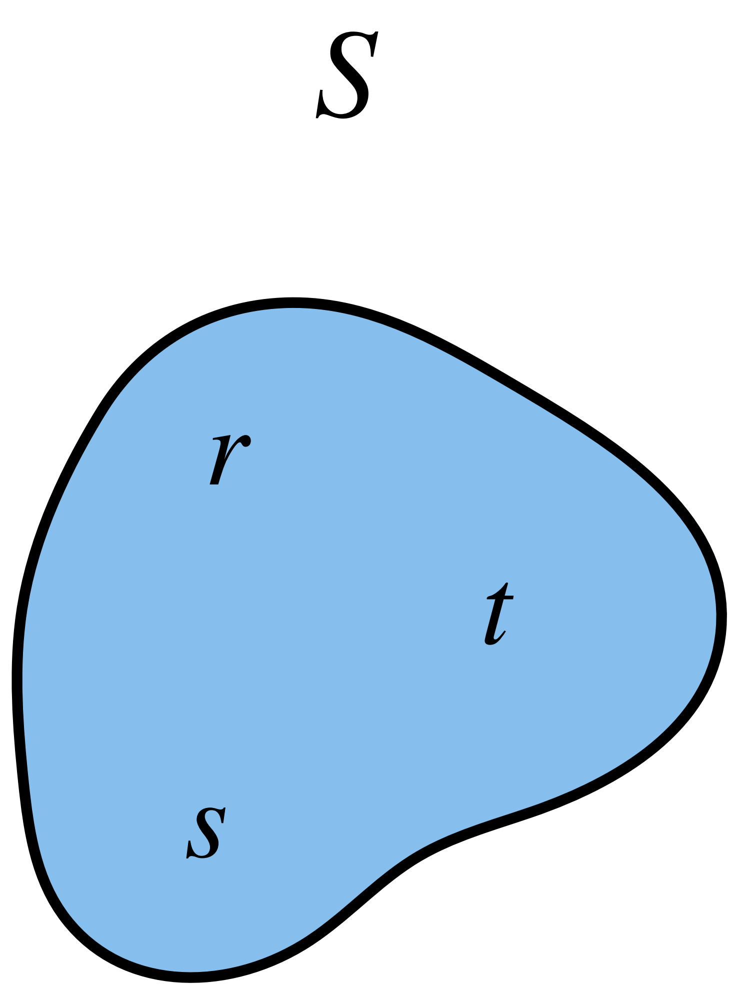

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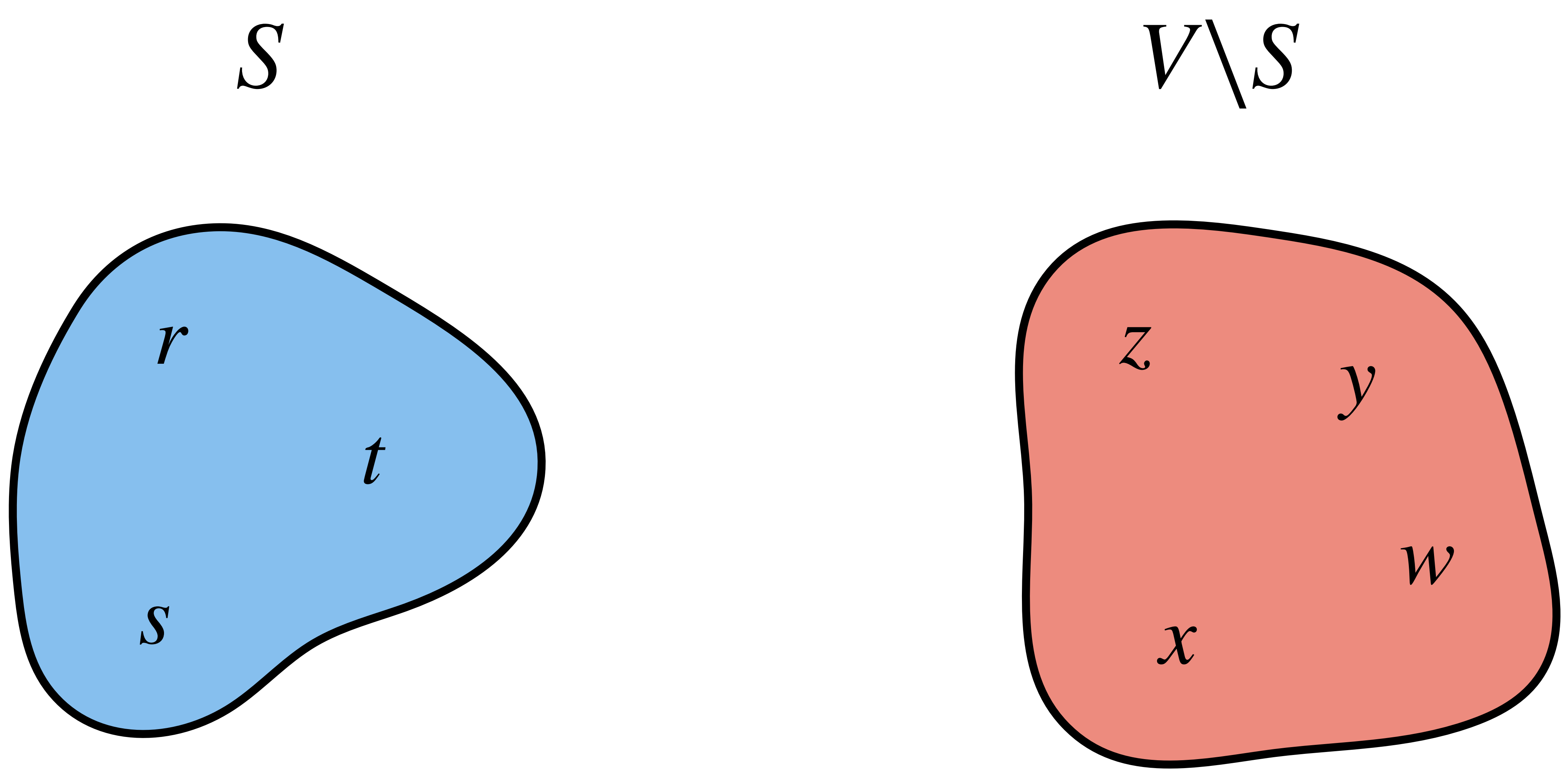

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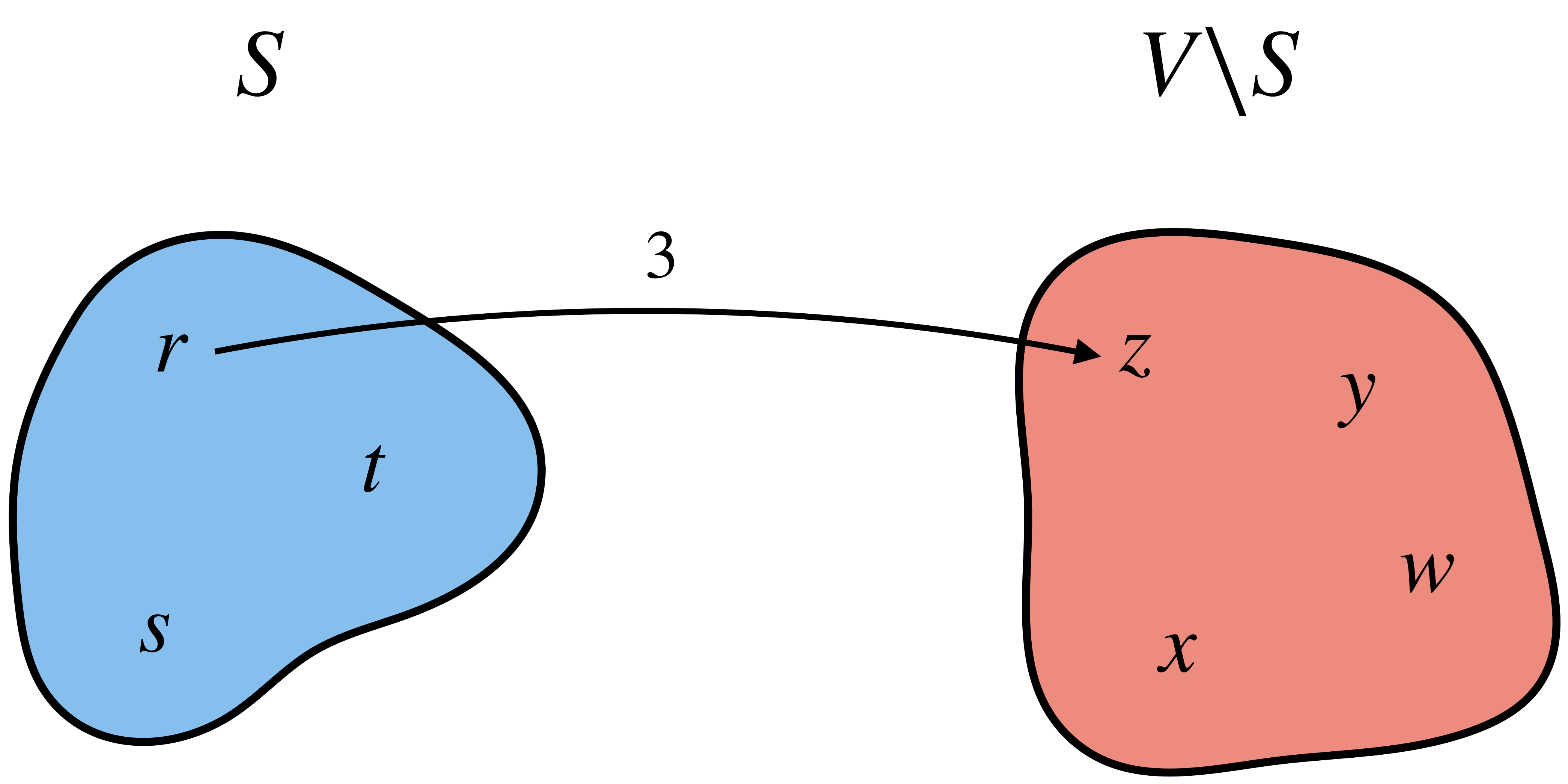

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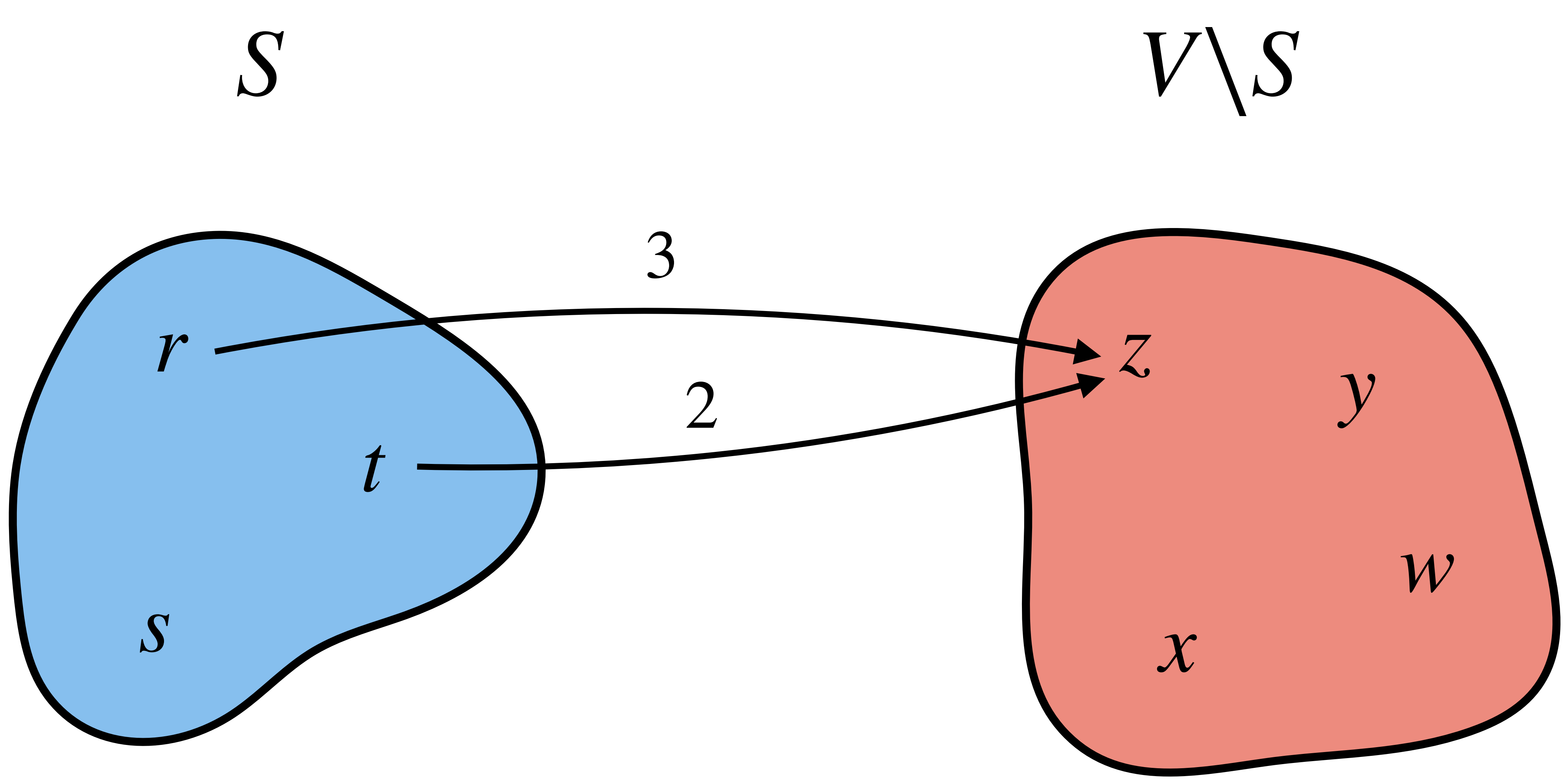

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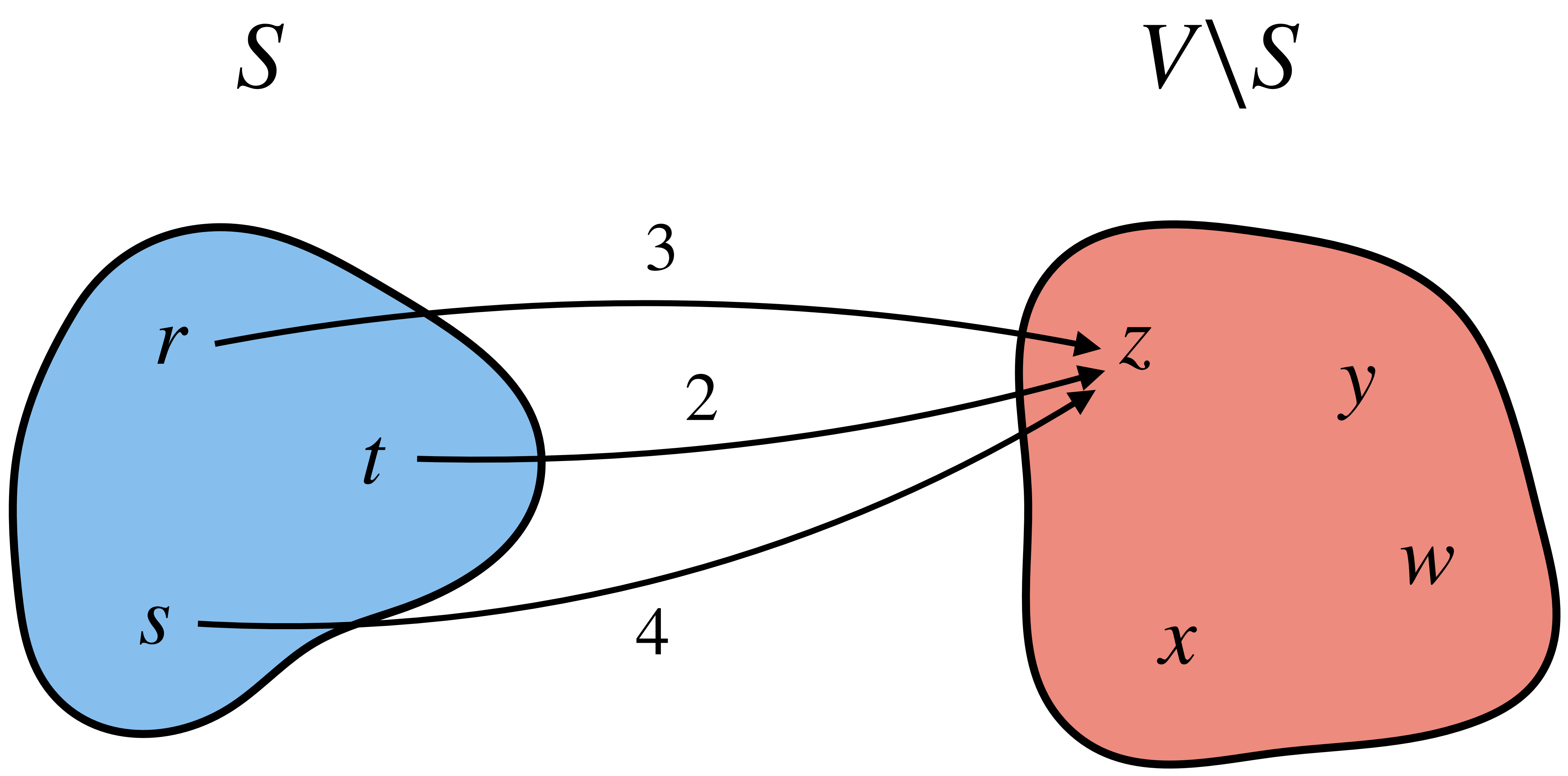

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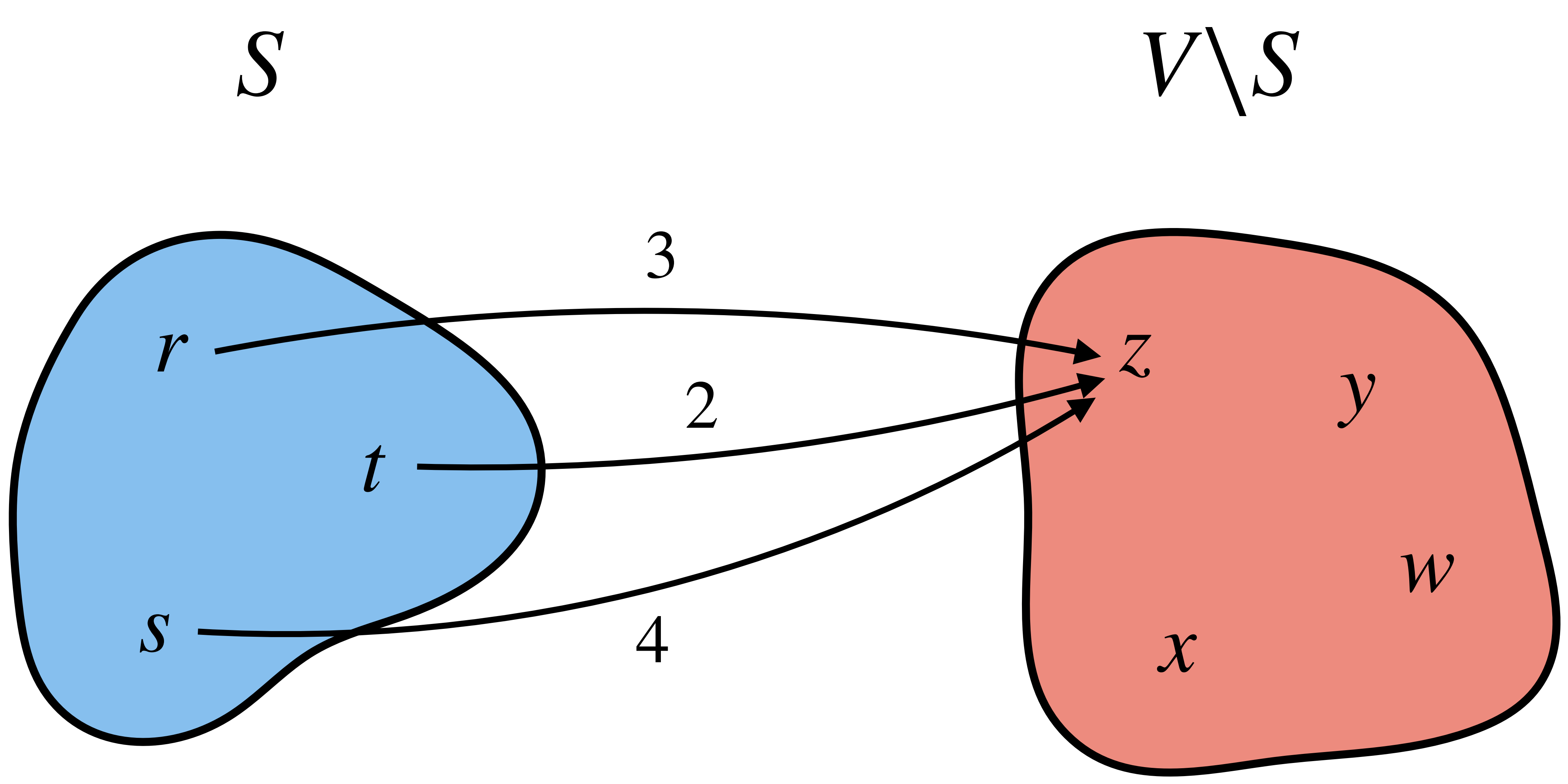

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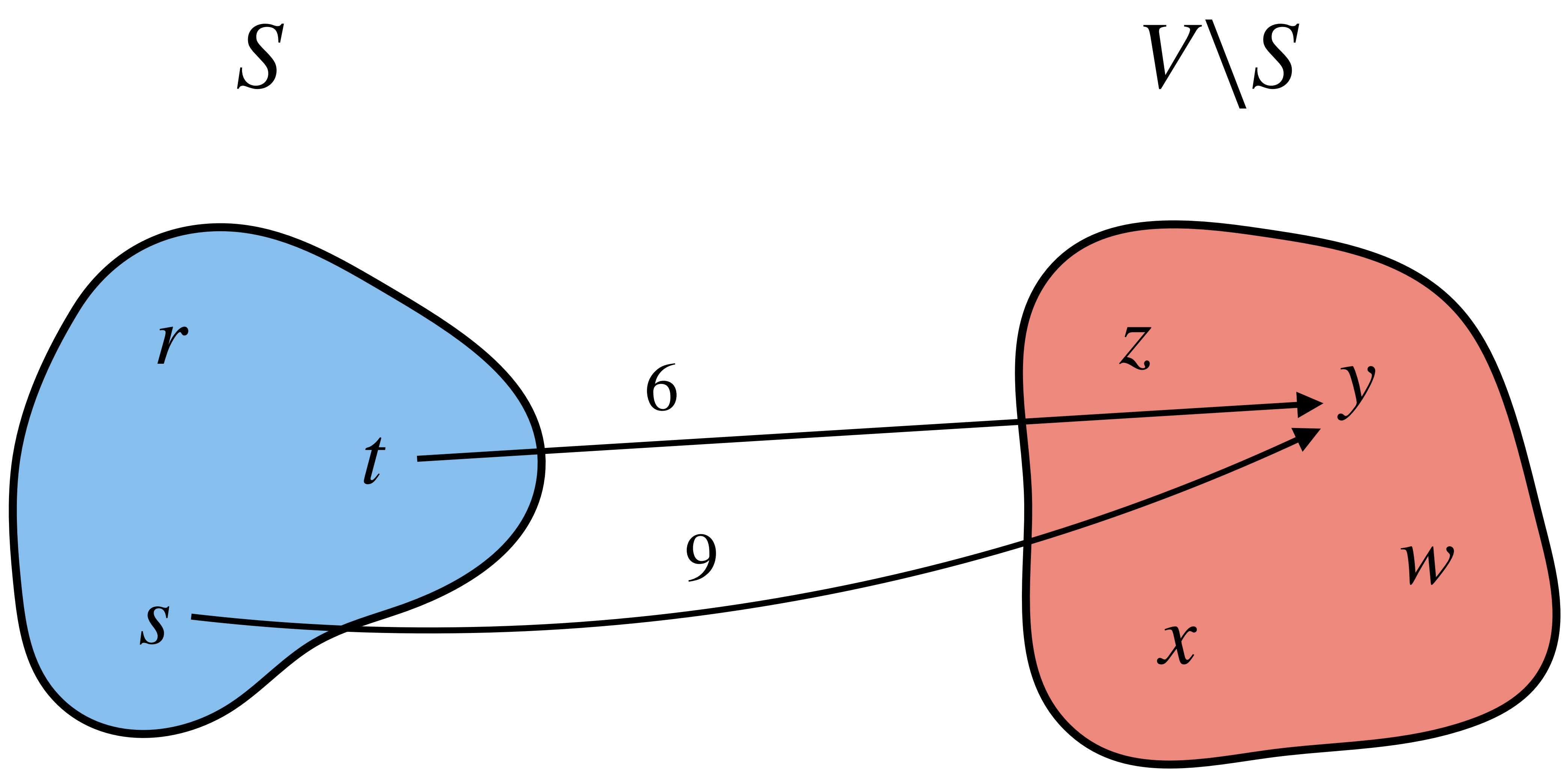

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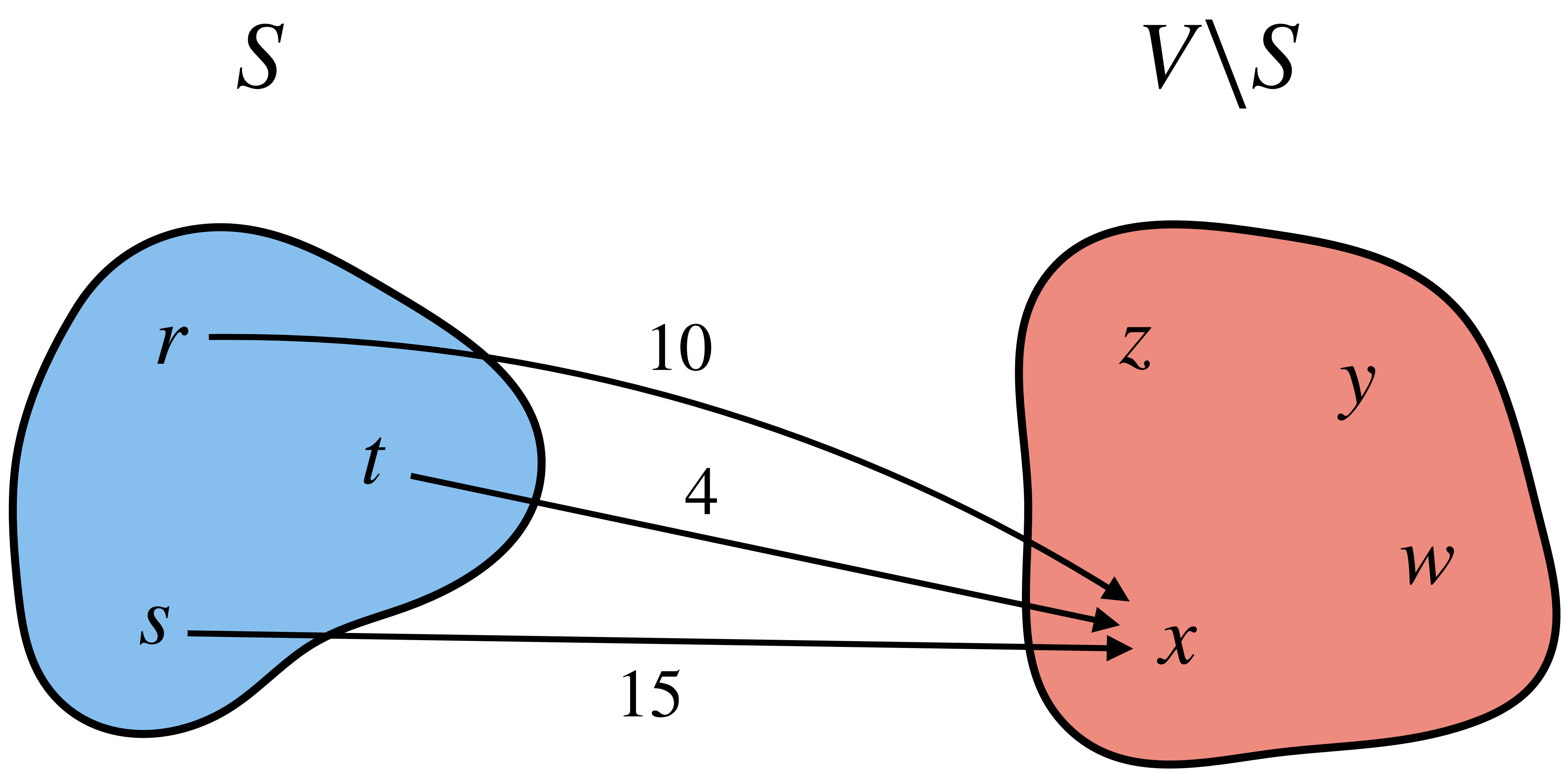

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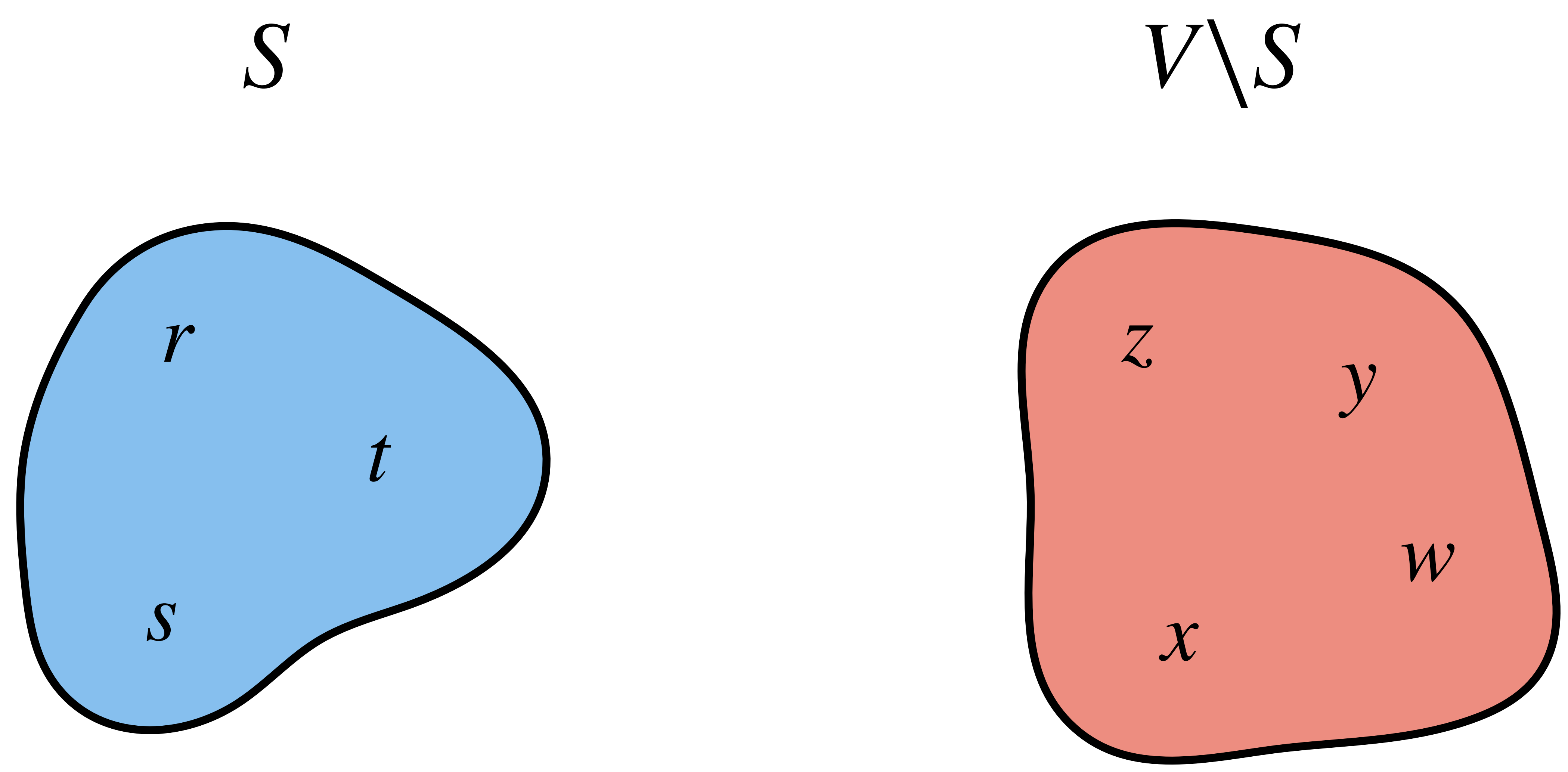

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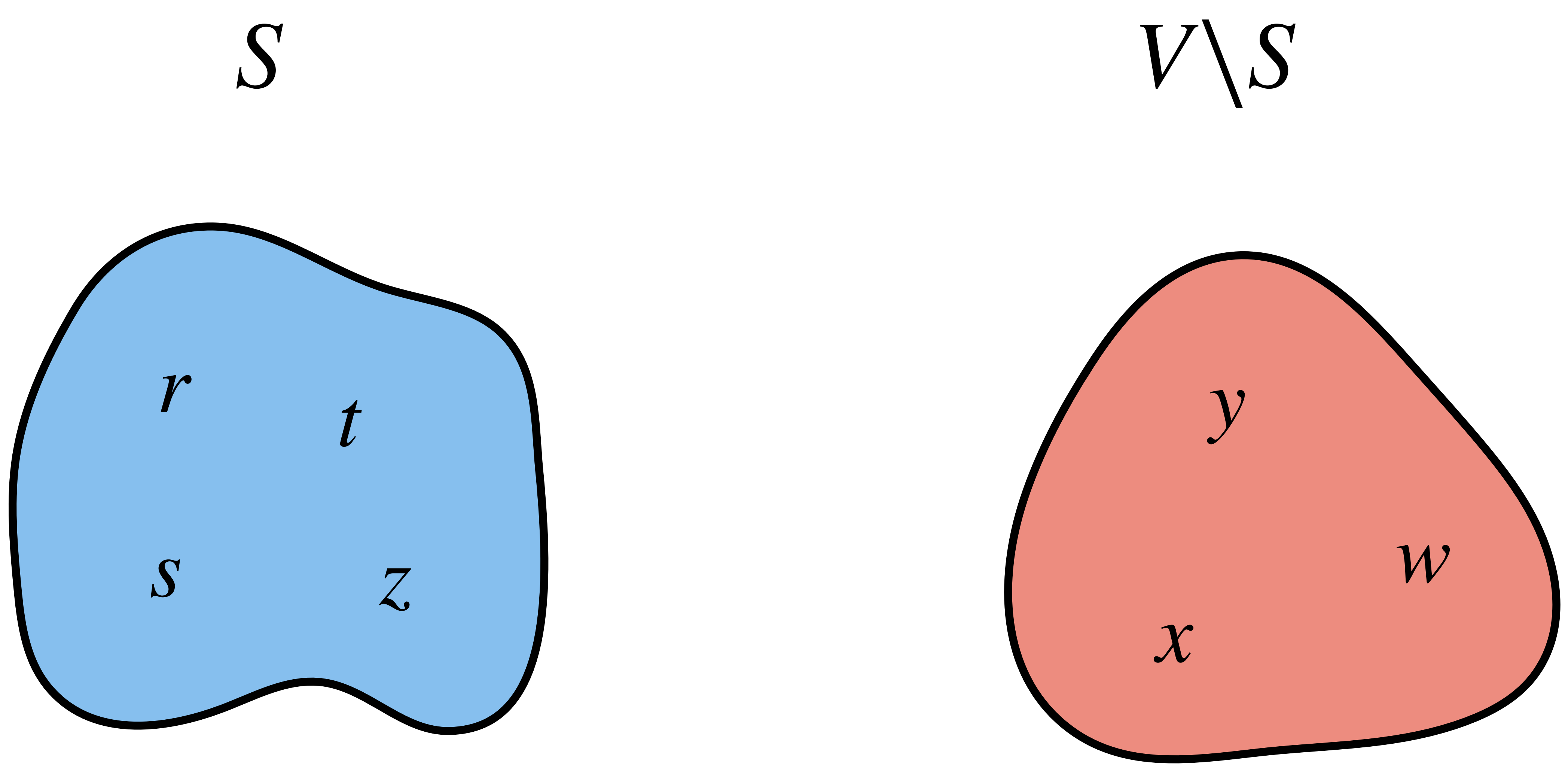

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Dijkstra's Algorithm: Sketch

Maintain a set of explored vertices S for which algorithm has found $d[u] = \delta(s, u)$:

Step 1: Initialise $S = \{s\}$, $d[s] = 0$.

Step 2: Choose an unexplored vertex v from $V(G) \backslash S$ which minimizes:

$$\pi[v] = \min_{(u,v)\in E,\ u\in S} d[u] + w(u, v)$$

Add v to S and set $d[v] = \Pi[v]$.

Step 3: Go to **Step 2** if it can be performed.

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Proof:

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Proof: **Basis Step:** When $|S| = 1$, statement is trivially true as $S = \{s\}$ and $d[s] = 0 = \delta(s, s)$.

Dijkstra's Algorithm: Correctness

Theorem: In the previous algorithm, for every $u \in S$, $d[u] = \delta(s, u)$.

Proof: Inductive Step: Assume the statement is true for some S such that $|S| \geq 1$.

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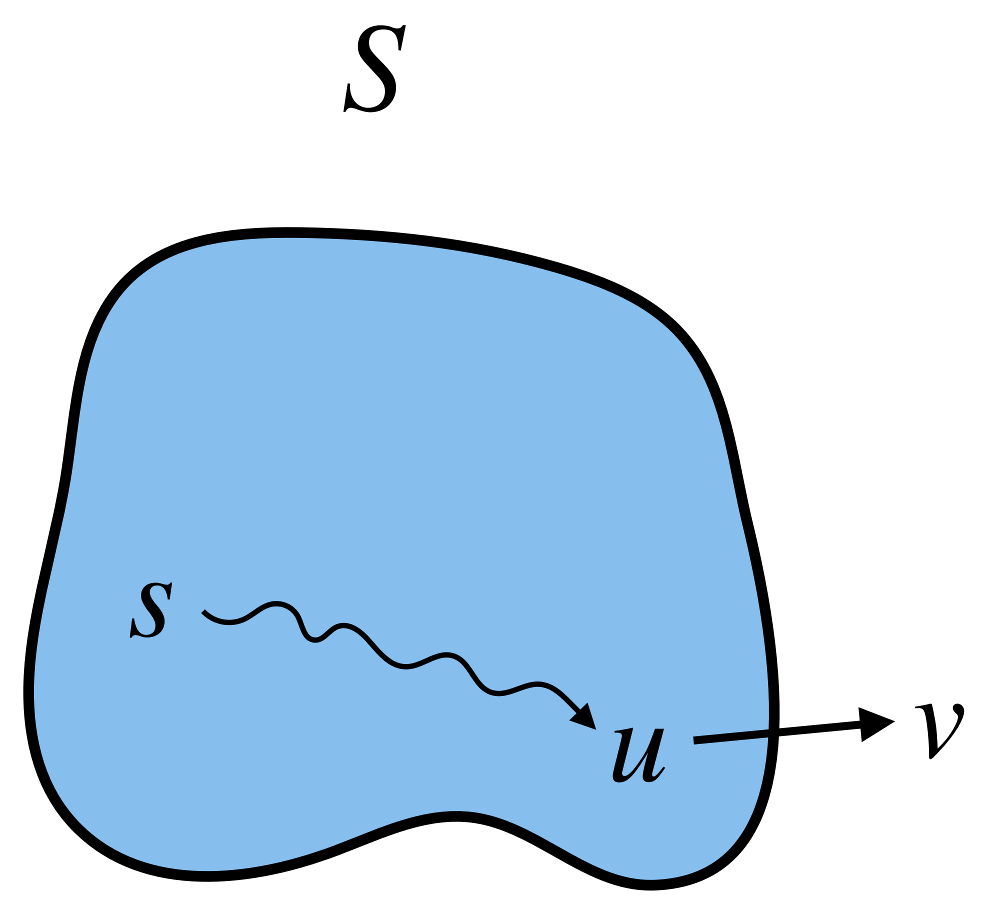

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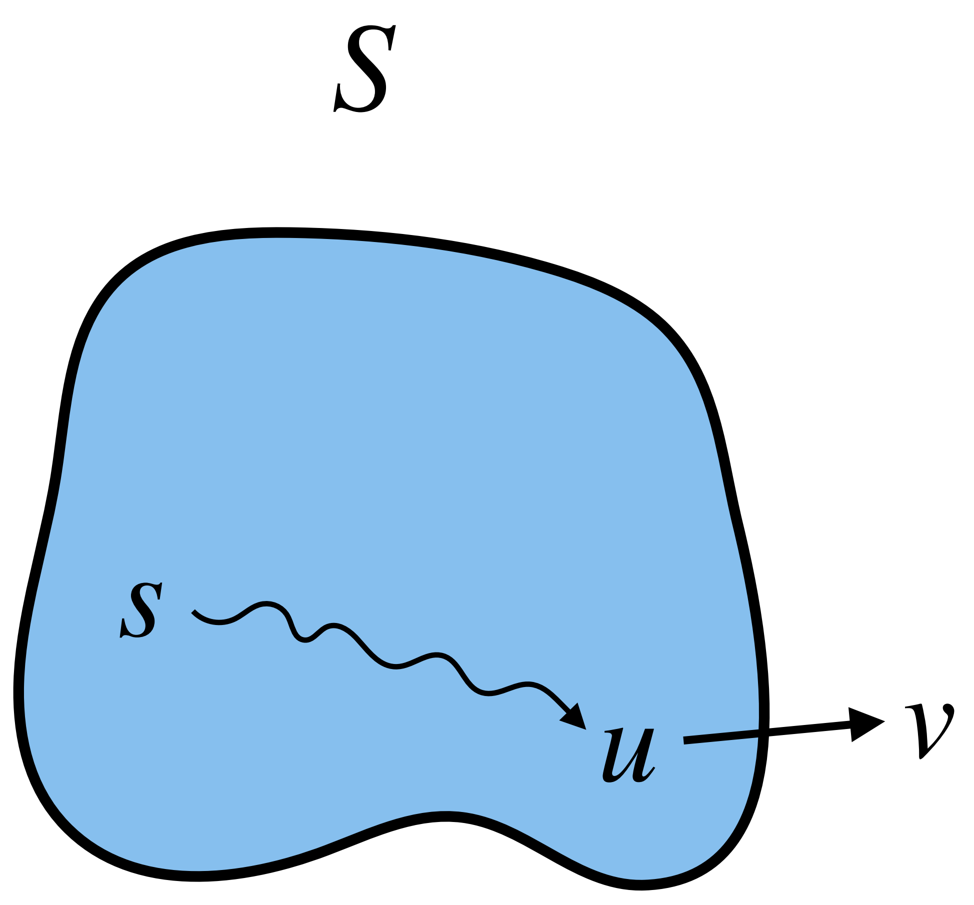

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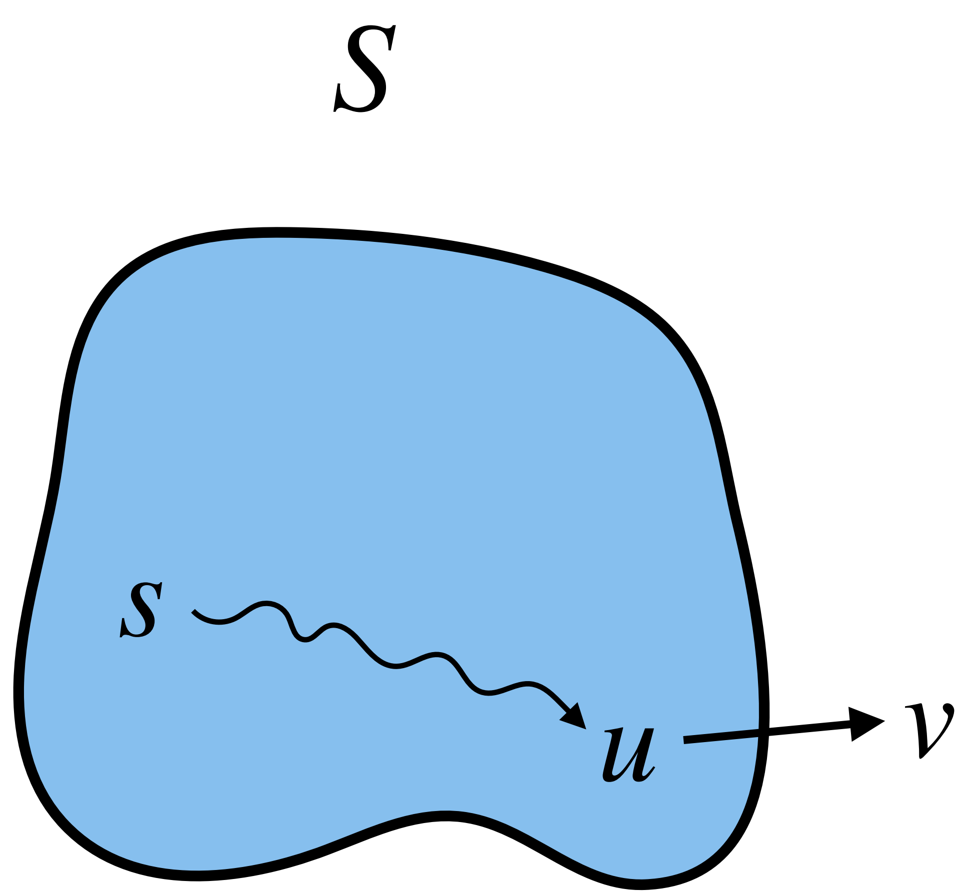

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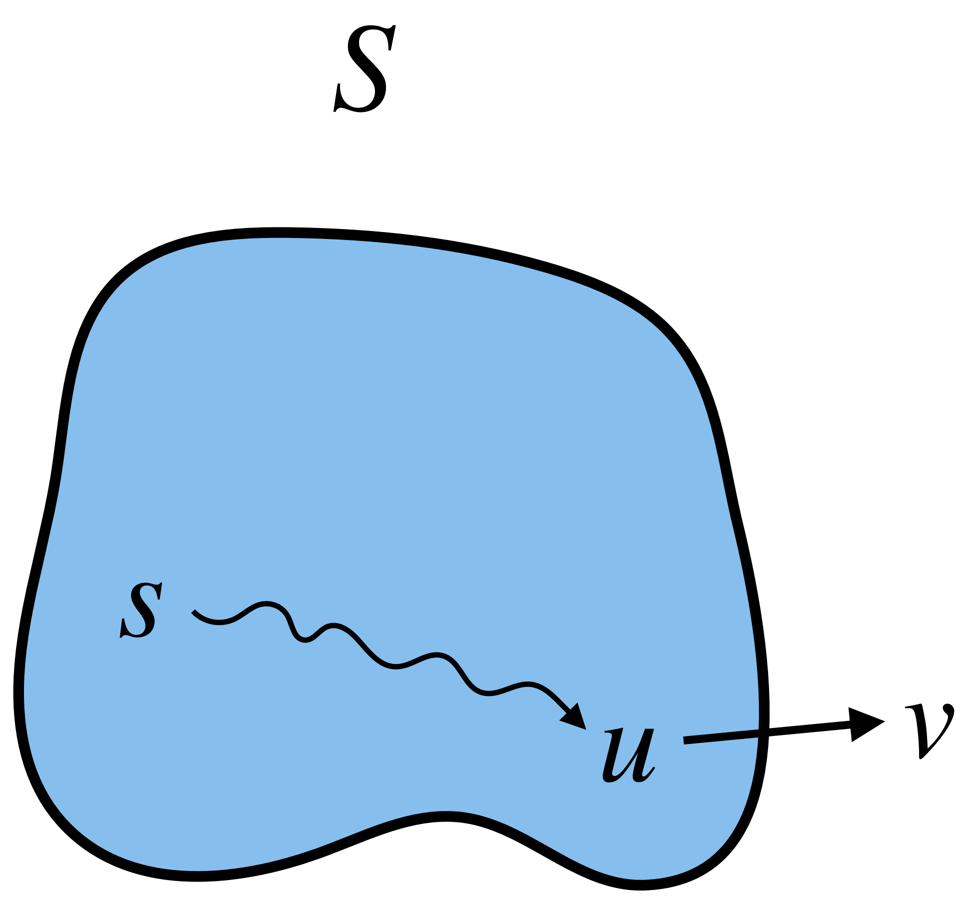

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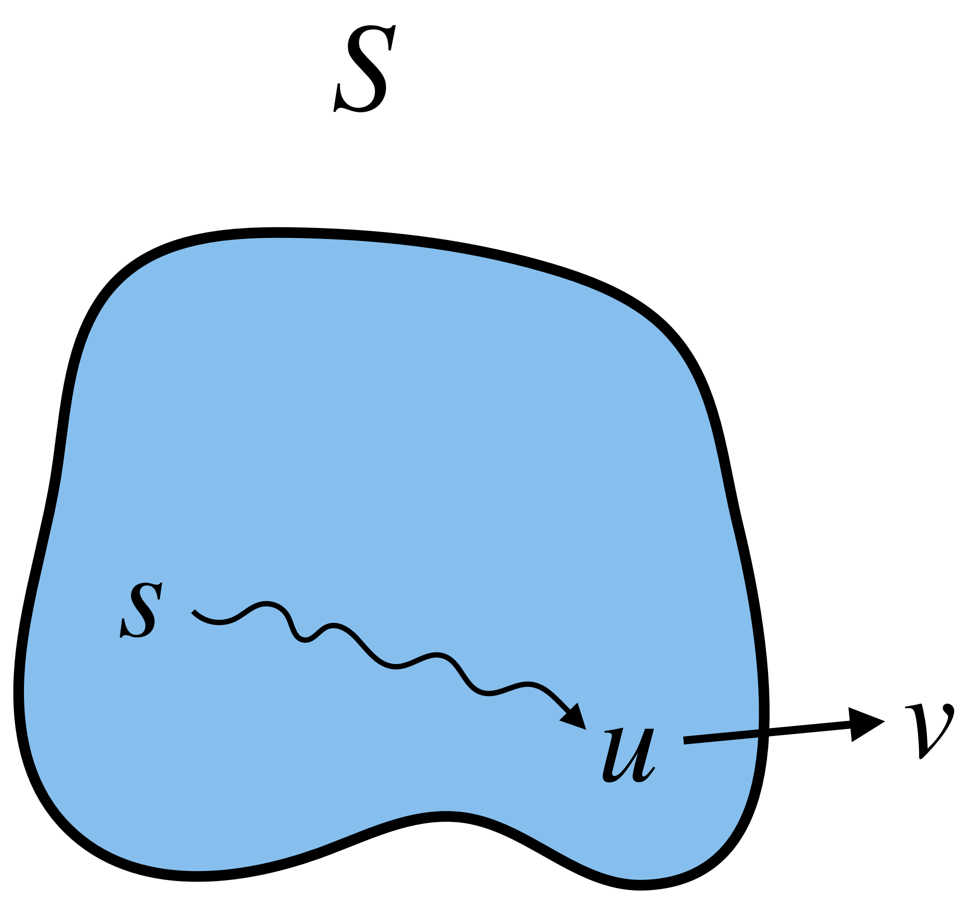

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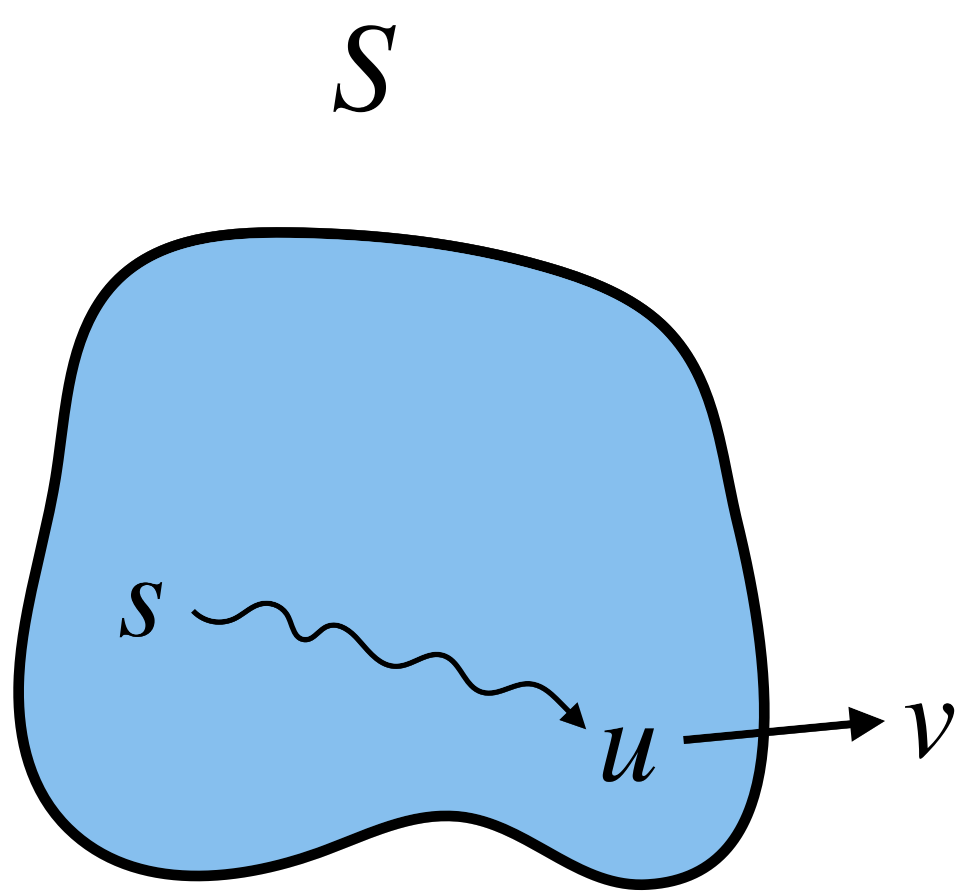

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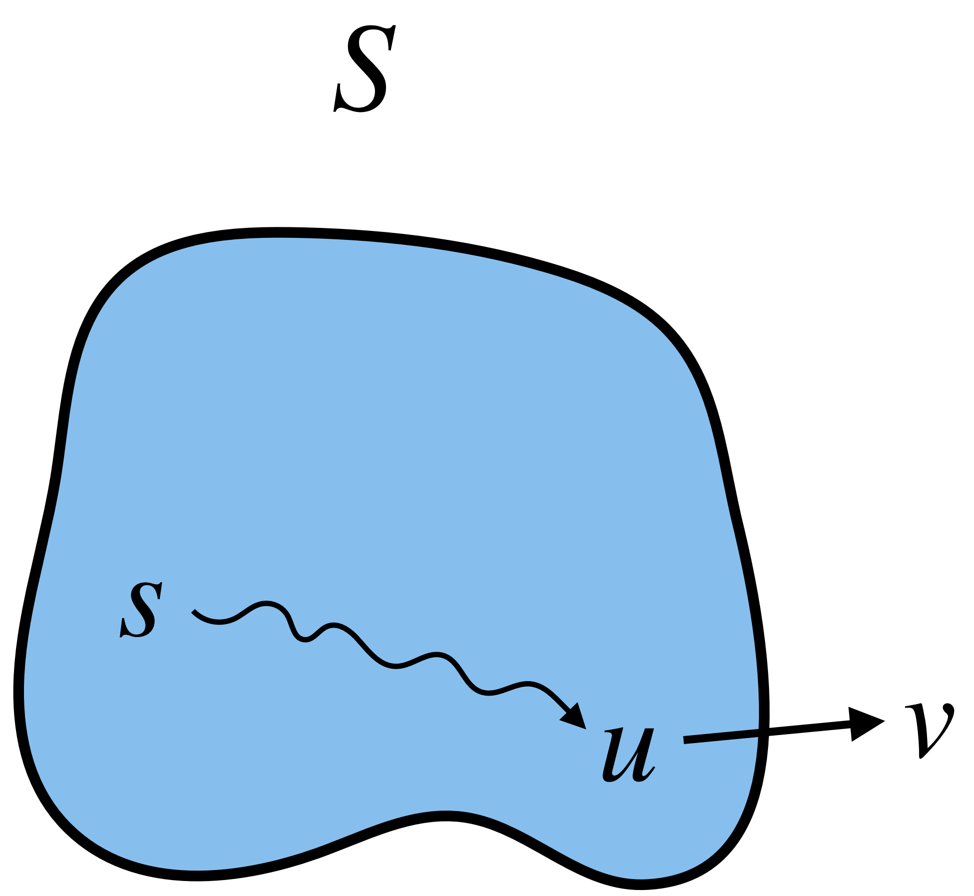

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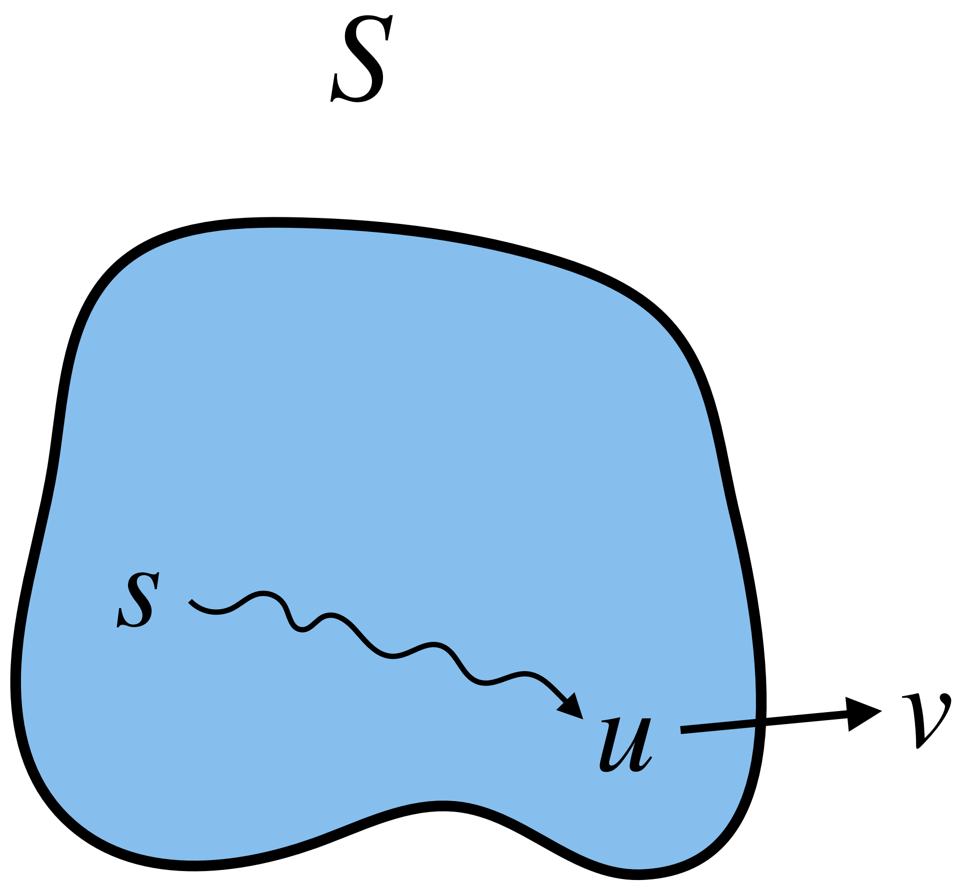

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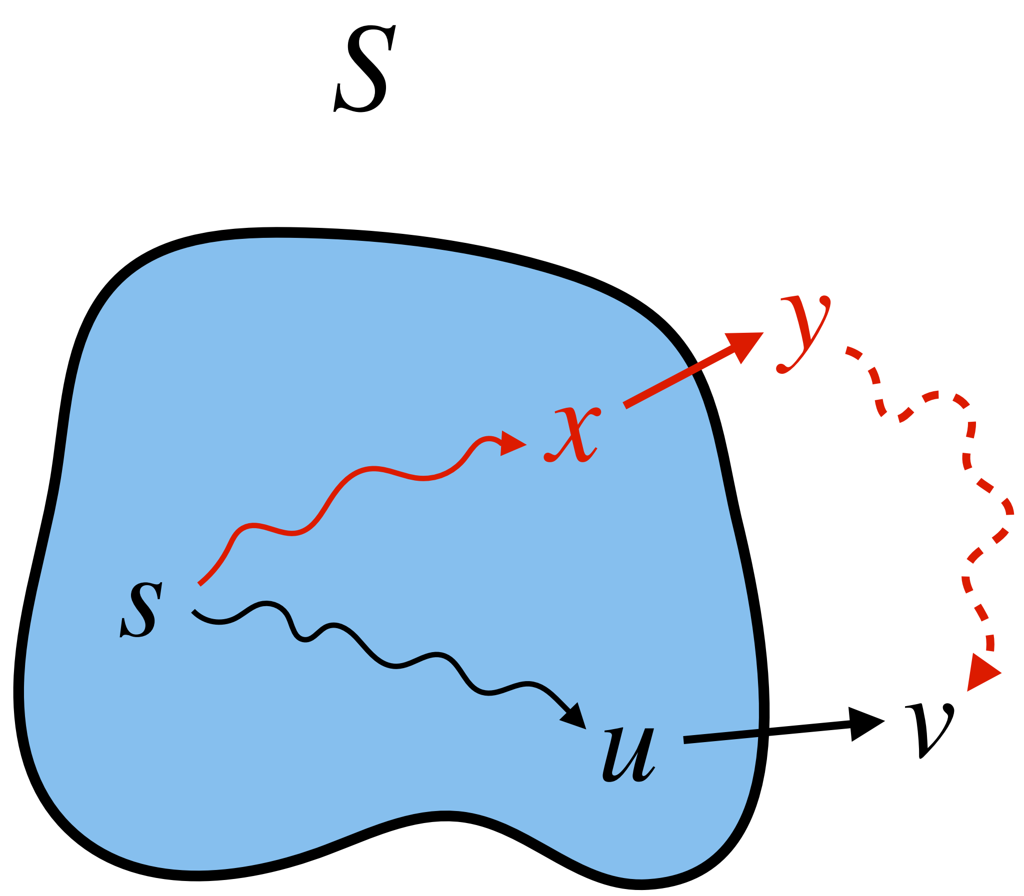

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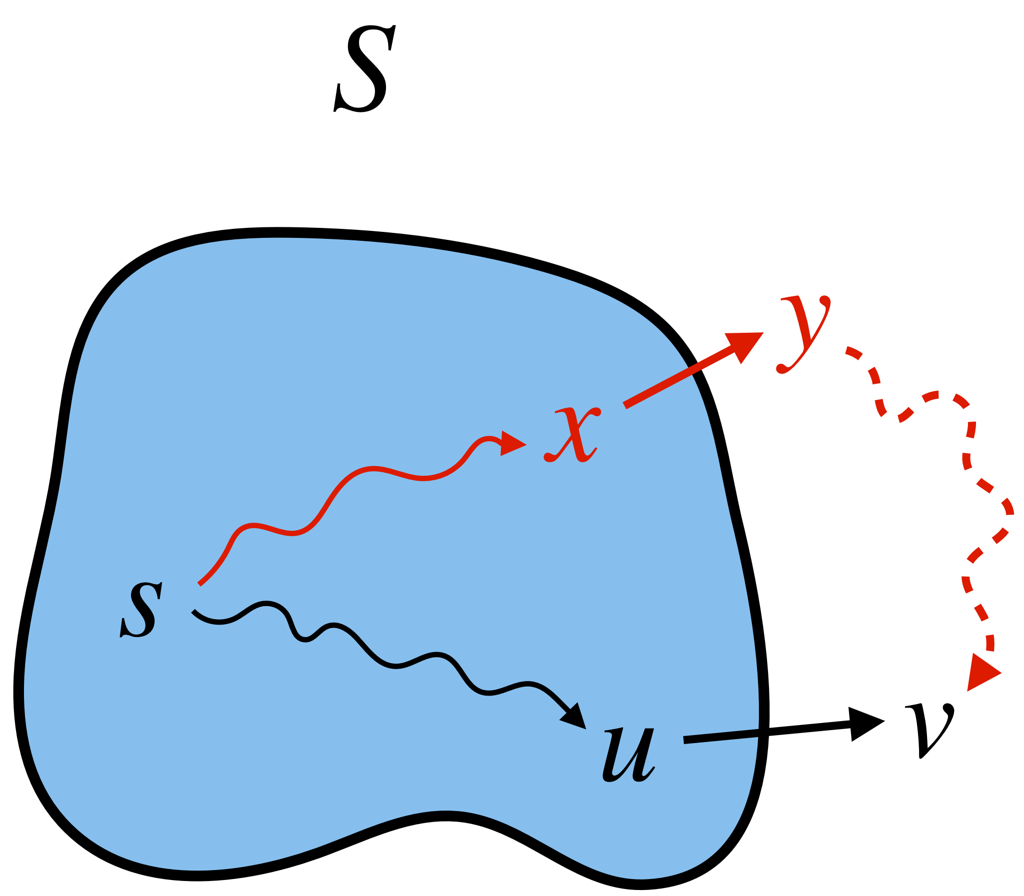

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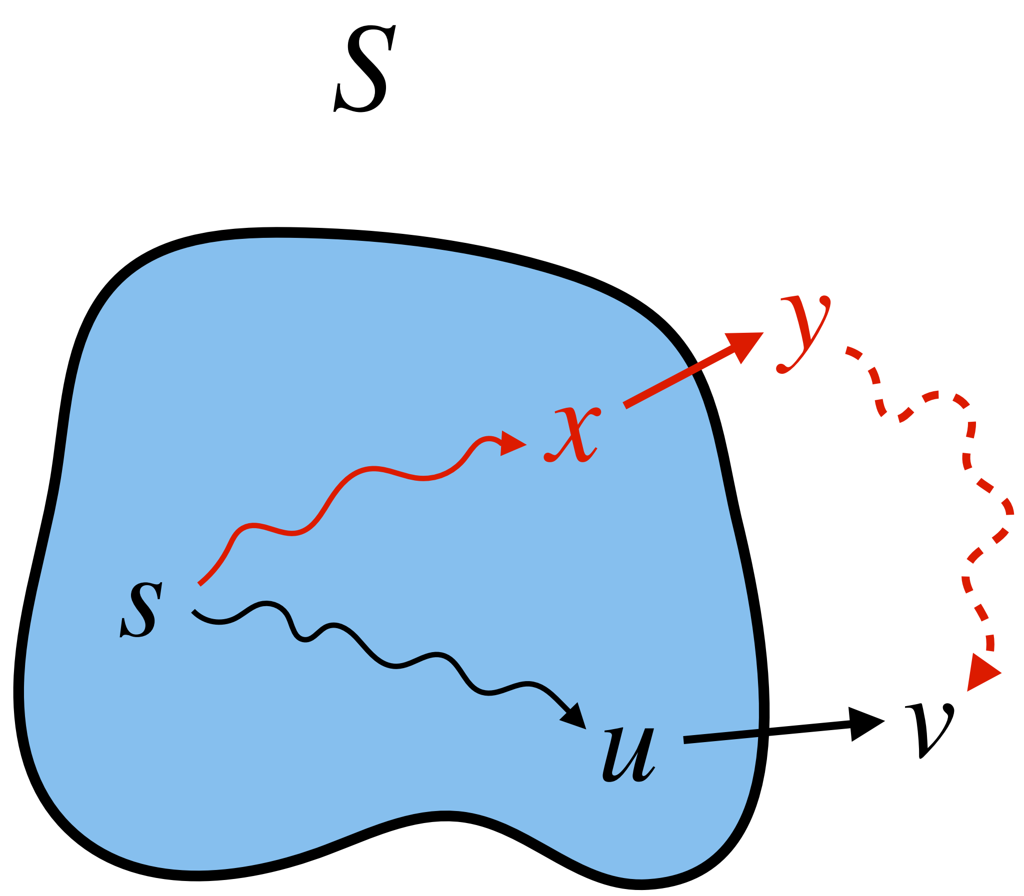

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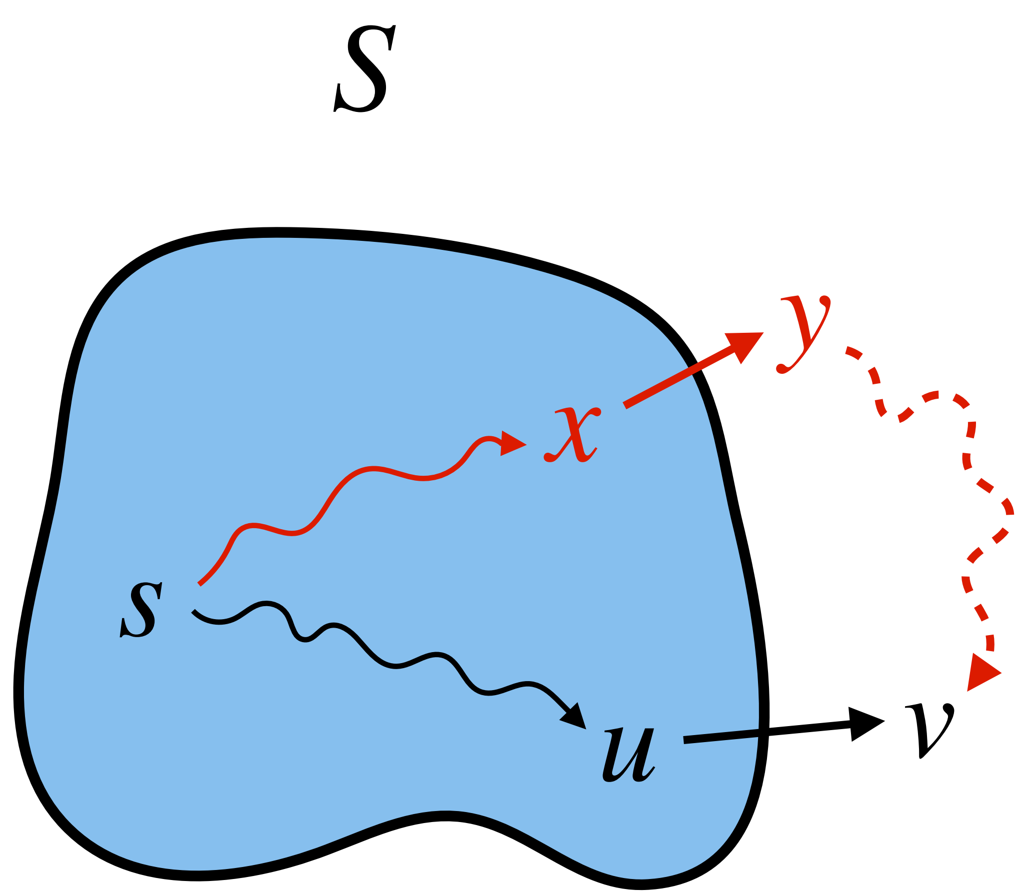

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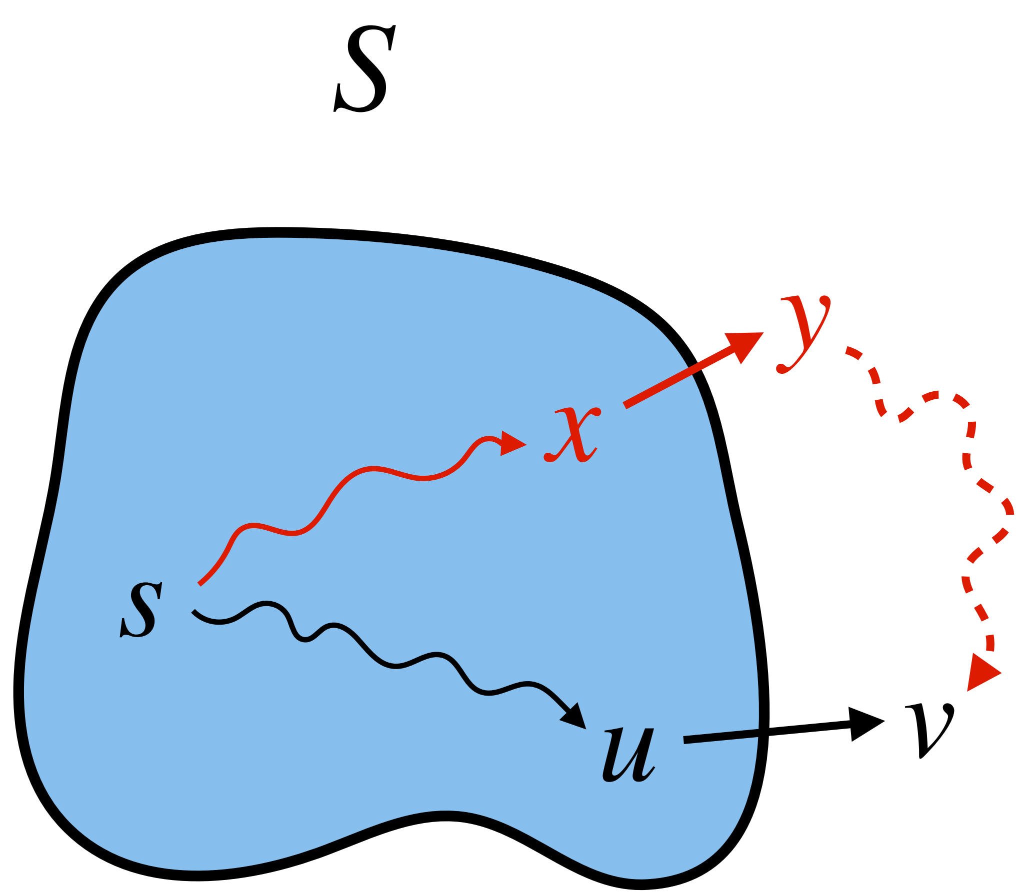

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Because Dijsktra chose v over y

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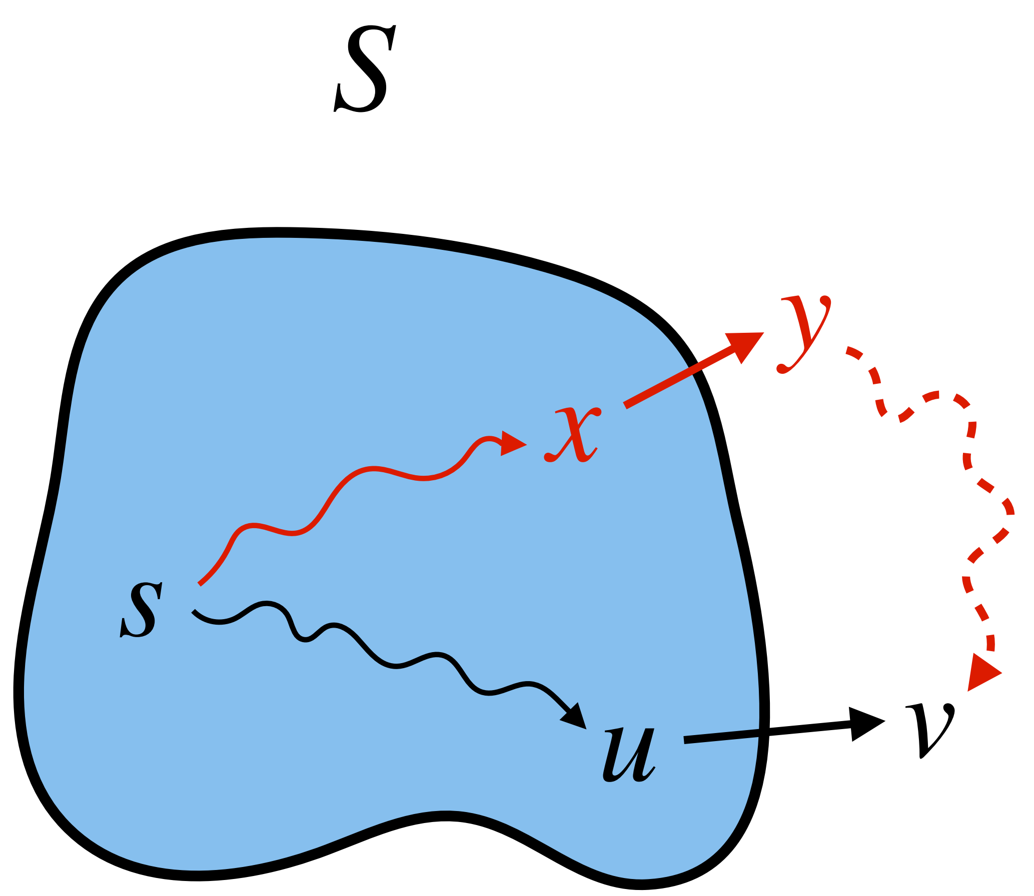

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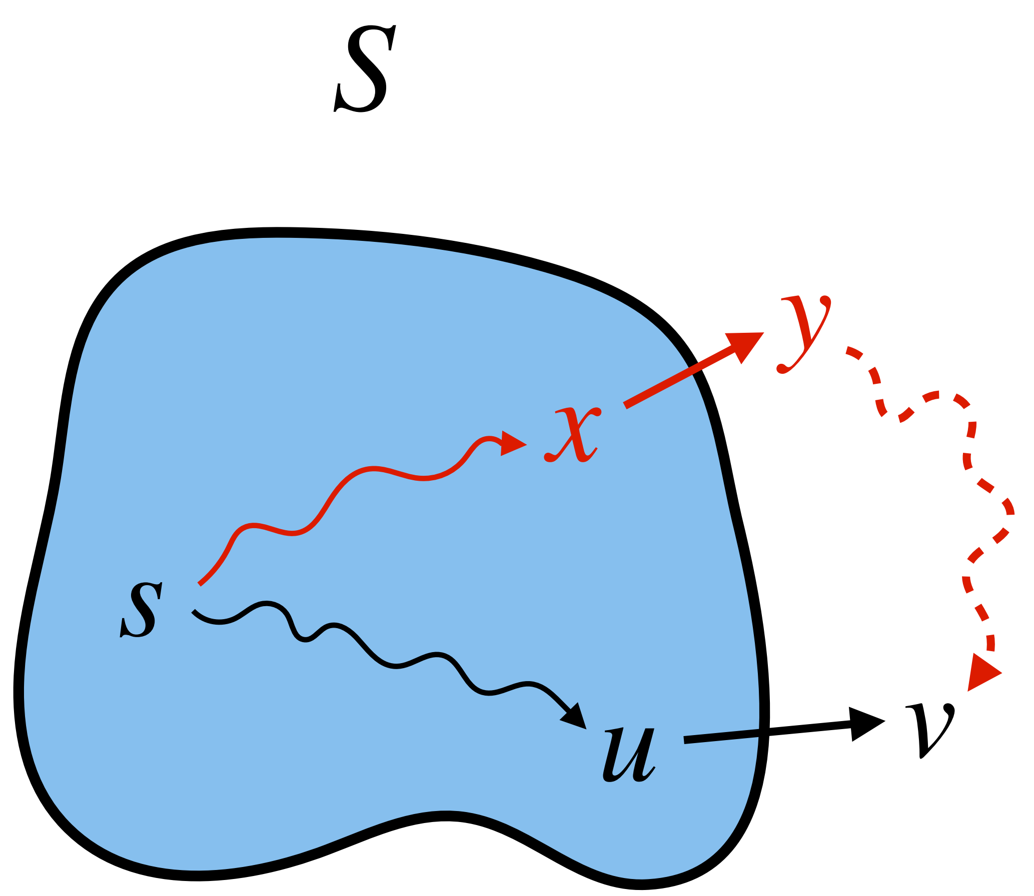

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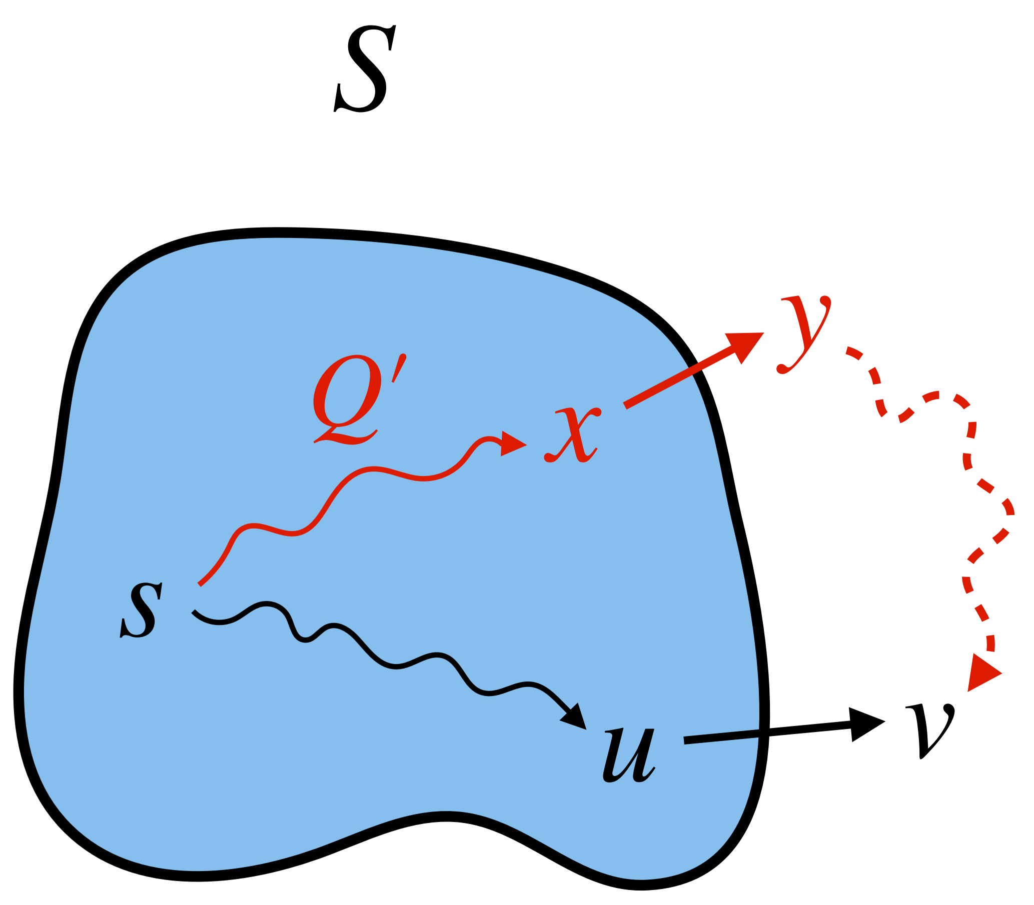

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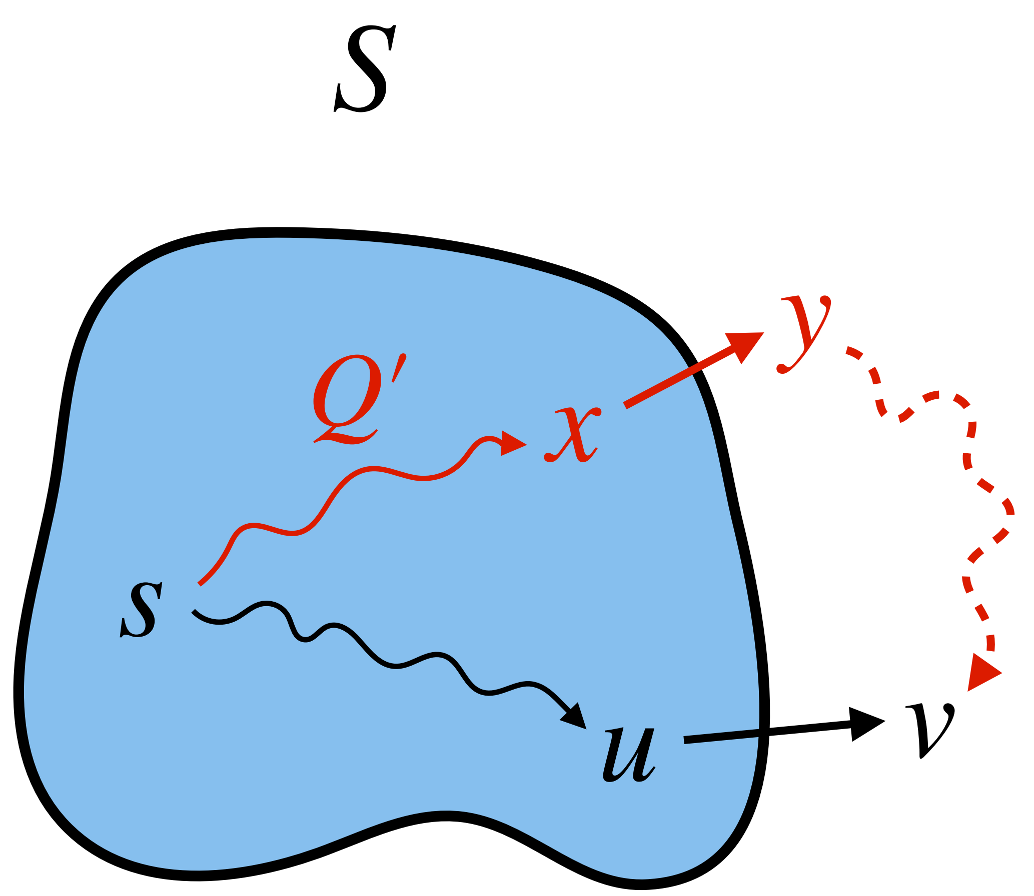

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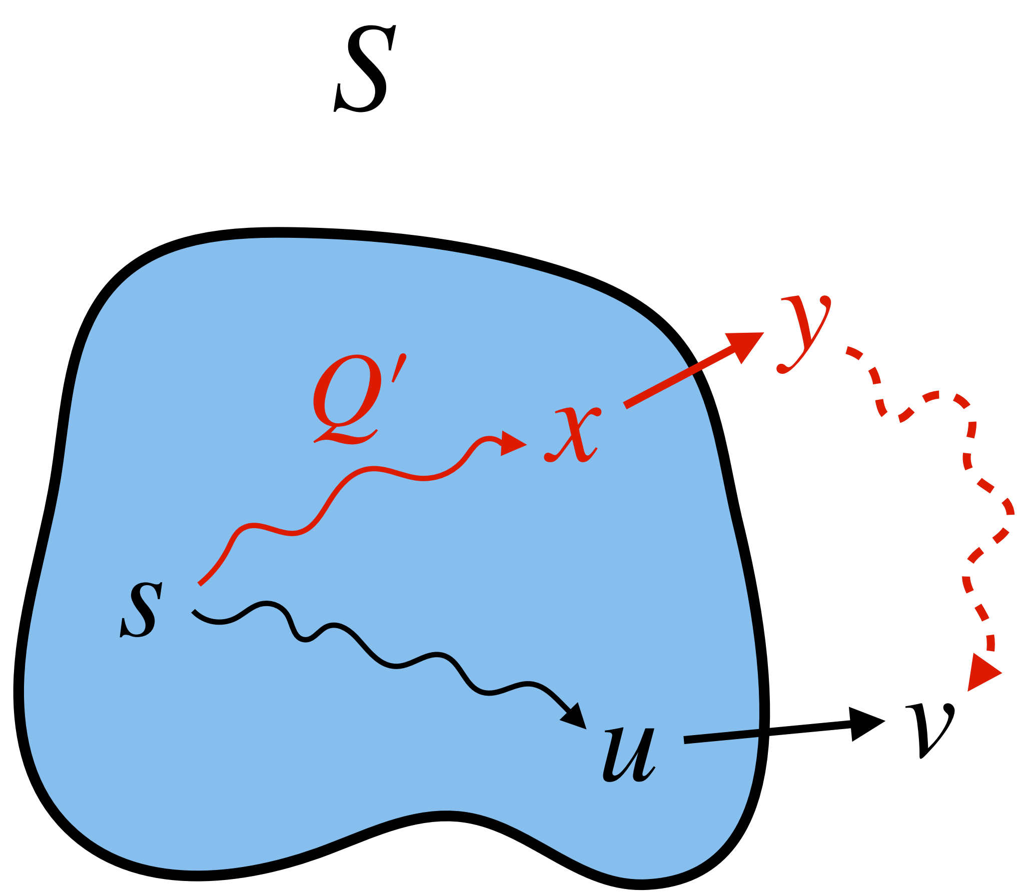

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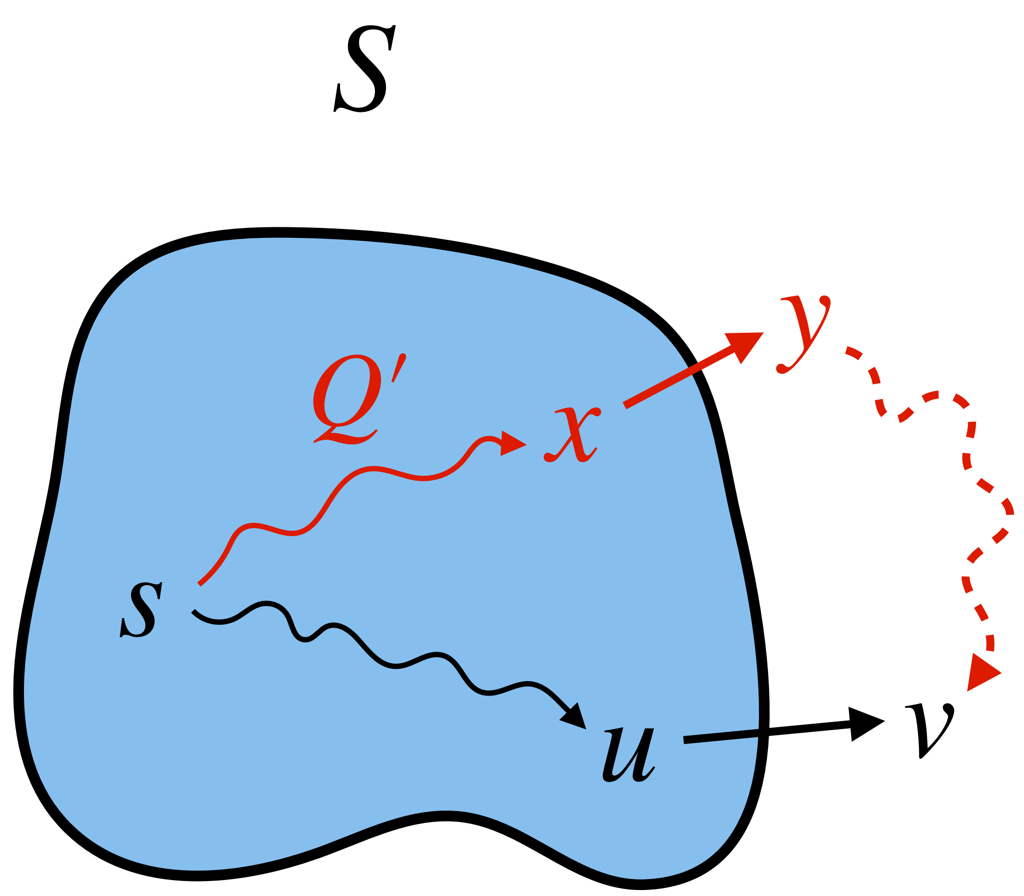

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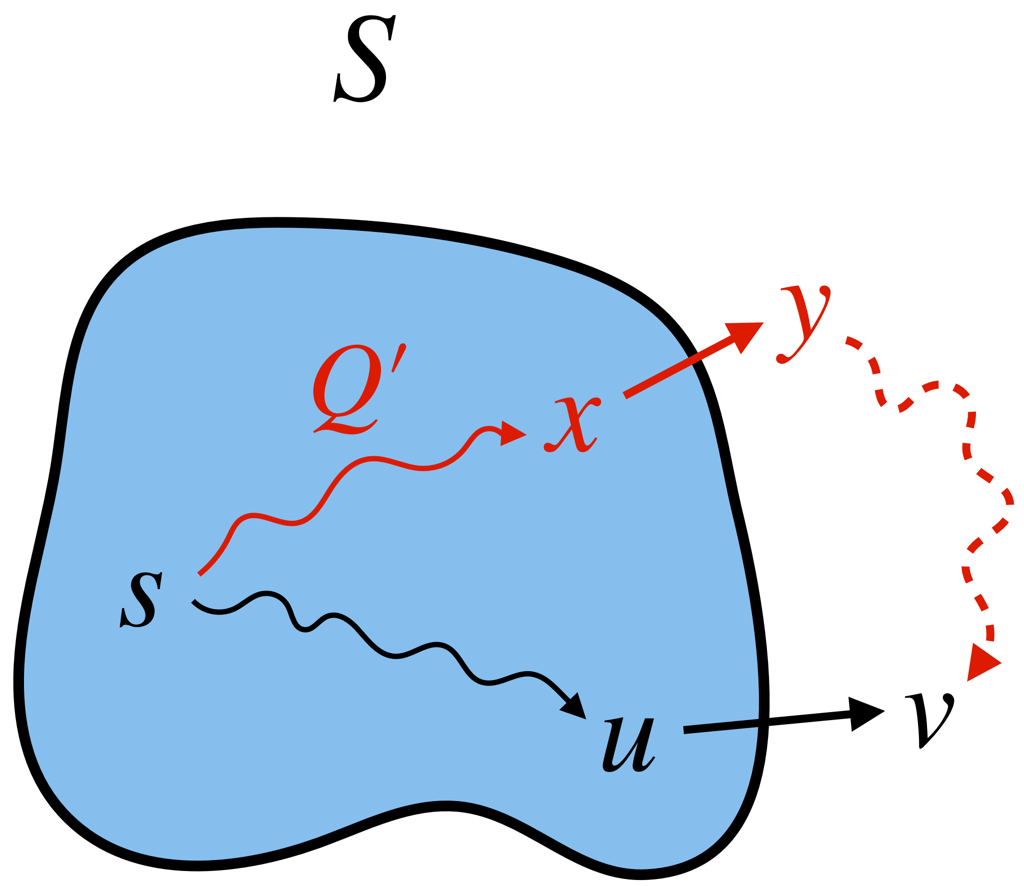

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Because y to v subpath in Q will have non-negative weights

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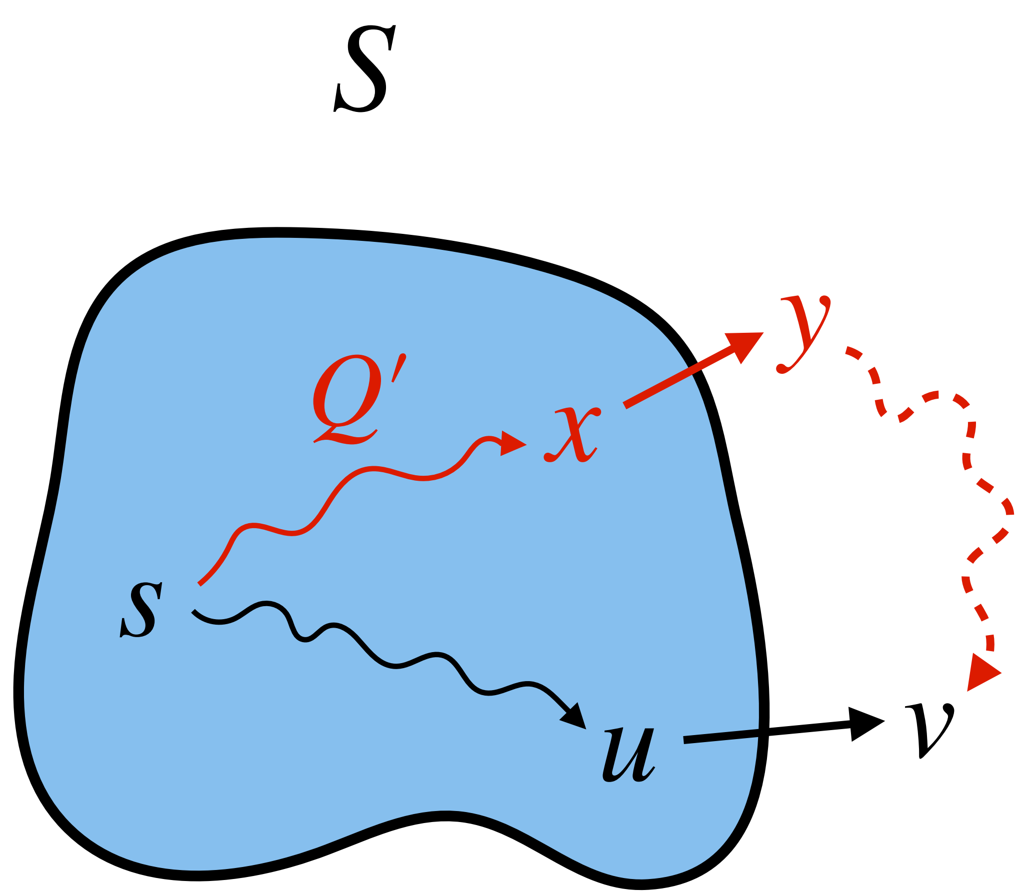

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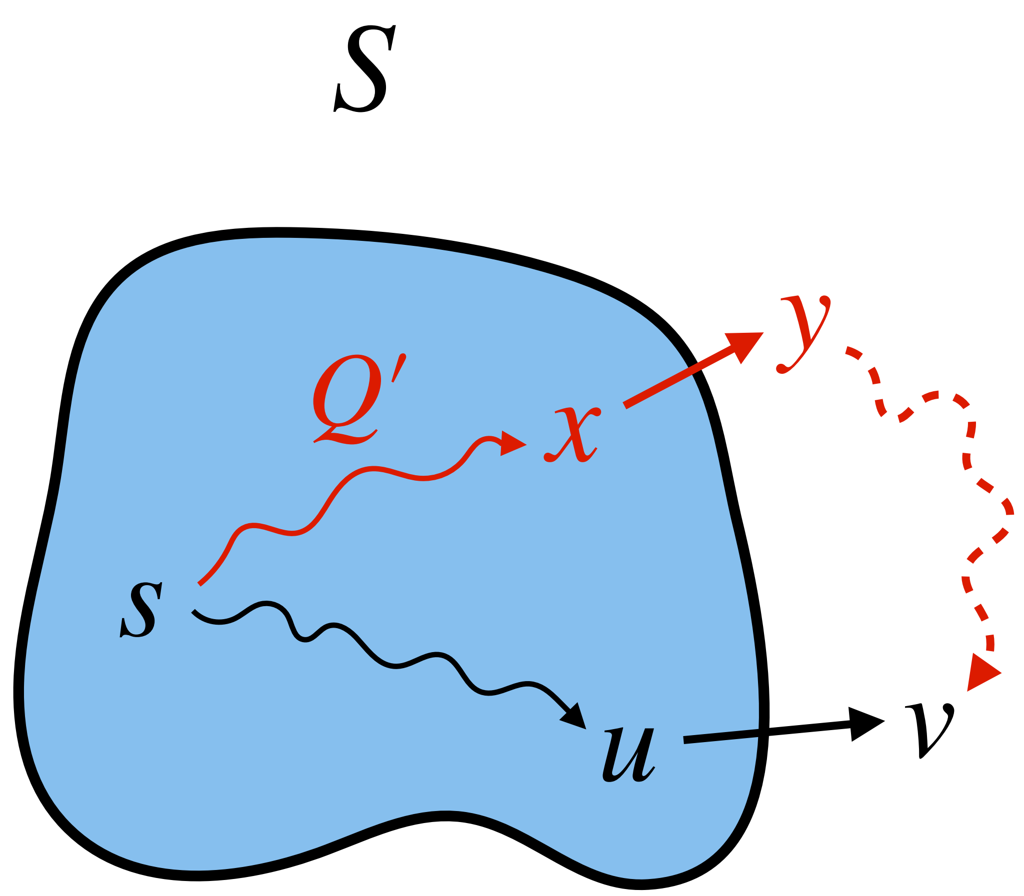

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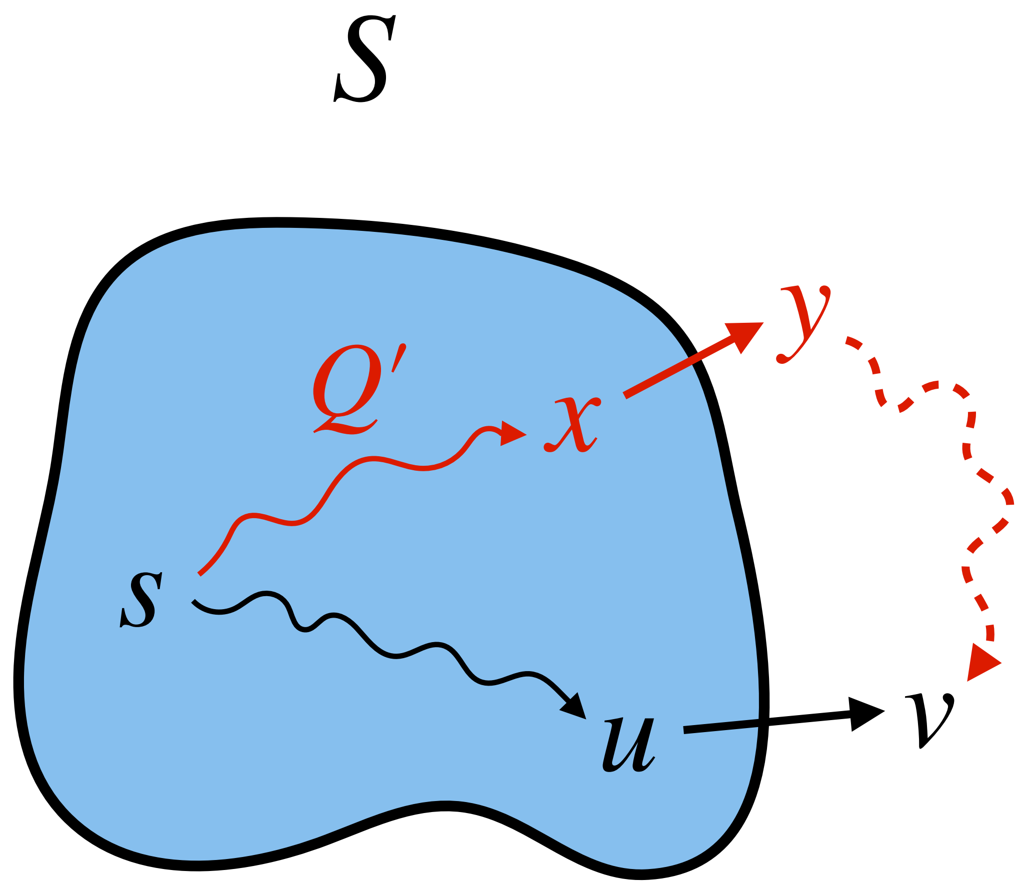

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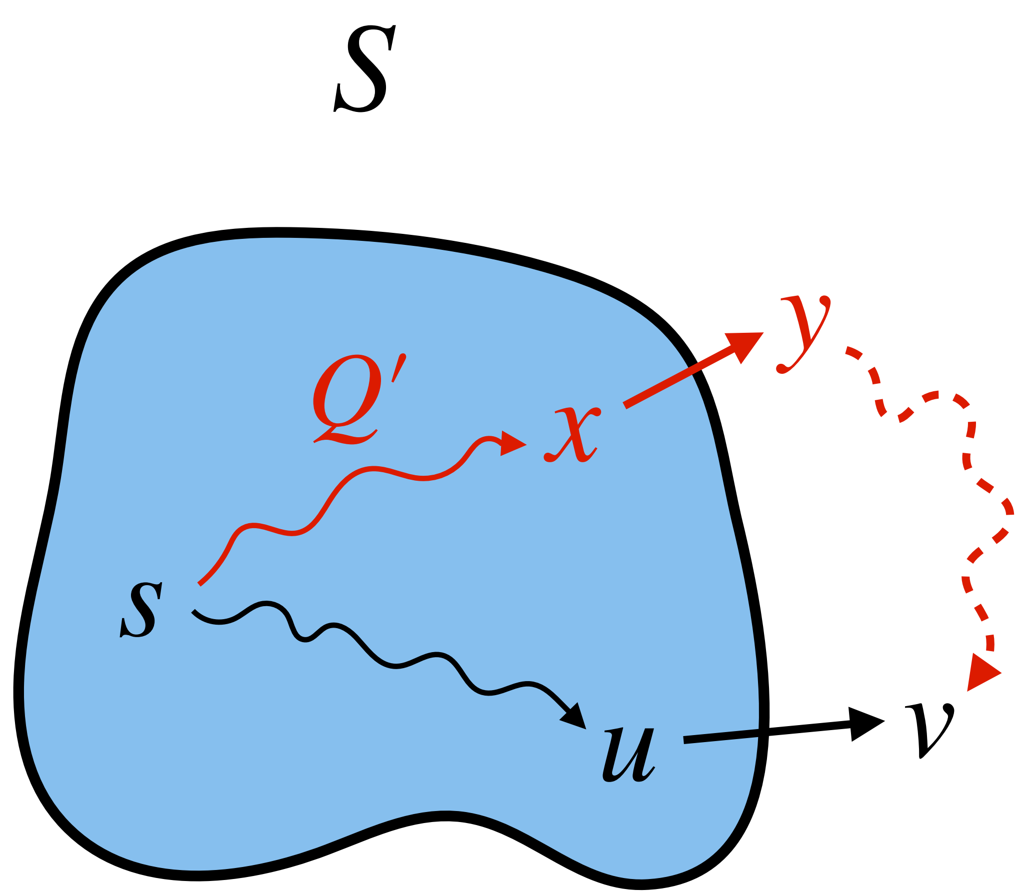

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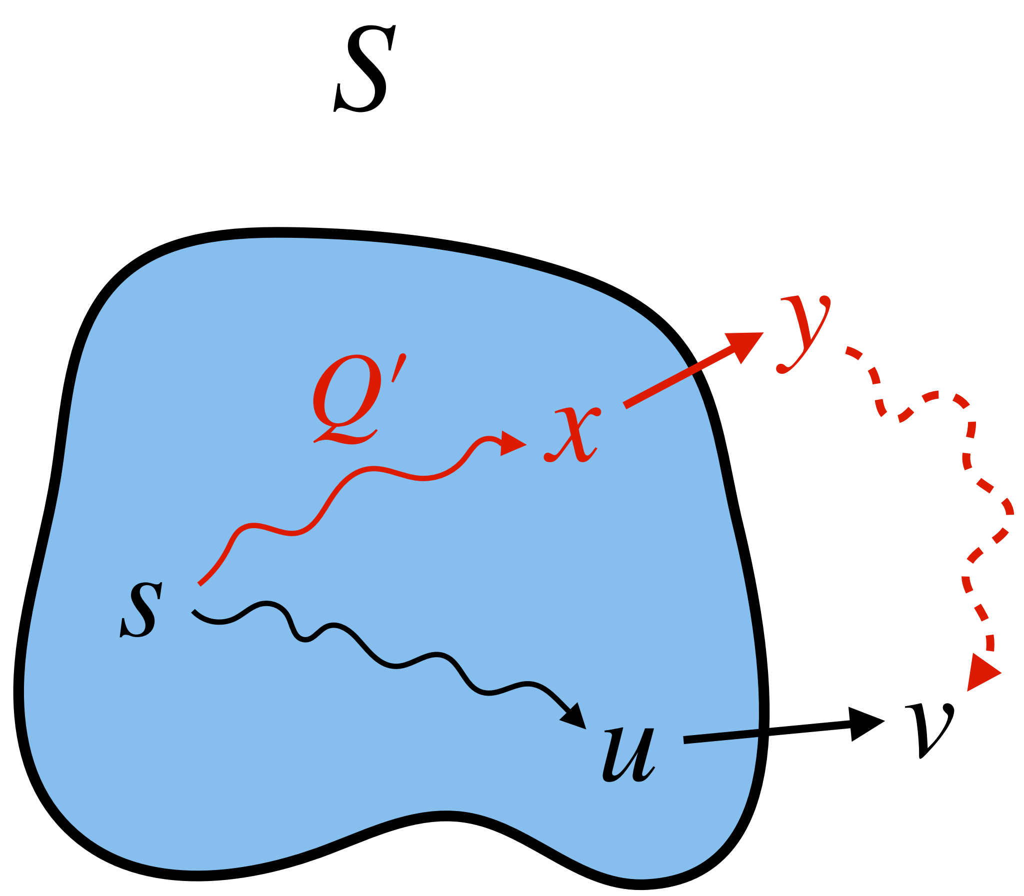

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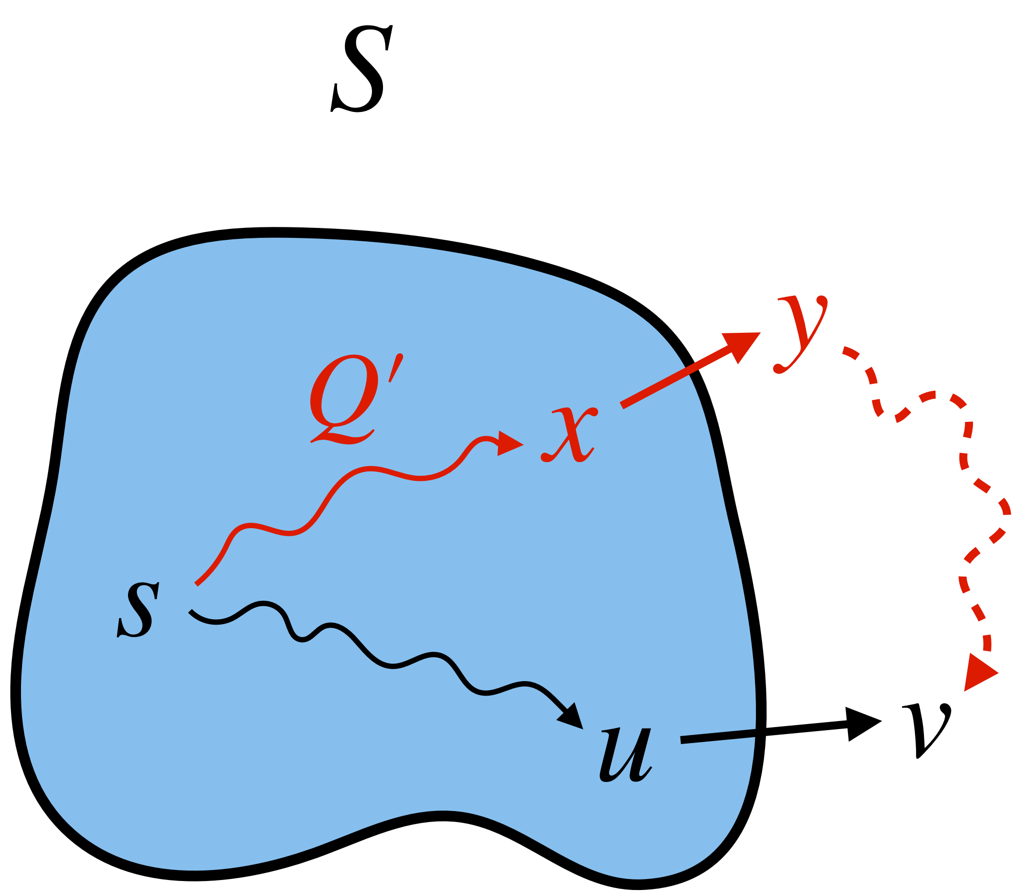

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Dijkstra's Algorithm: Sketch

Maintain a set of explored vertices S for which algorithm has found $d[u] = \delta(s, u)$:

Step 1: Initialise $S = \{s\}$, $d[s] = 0$.

Step 2: Choose an unexplored vertex v from $V \backslash S$ which minimizes:

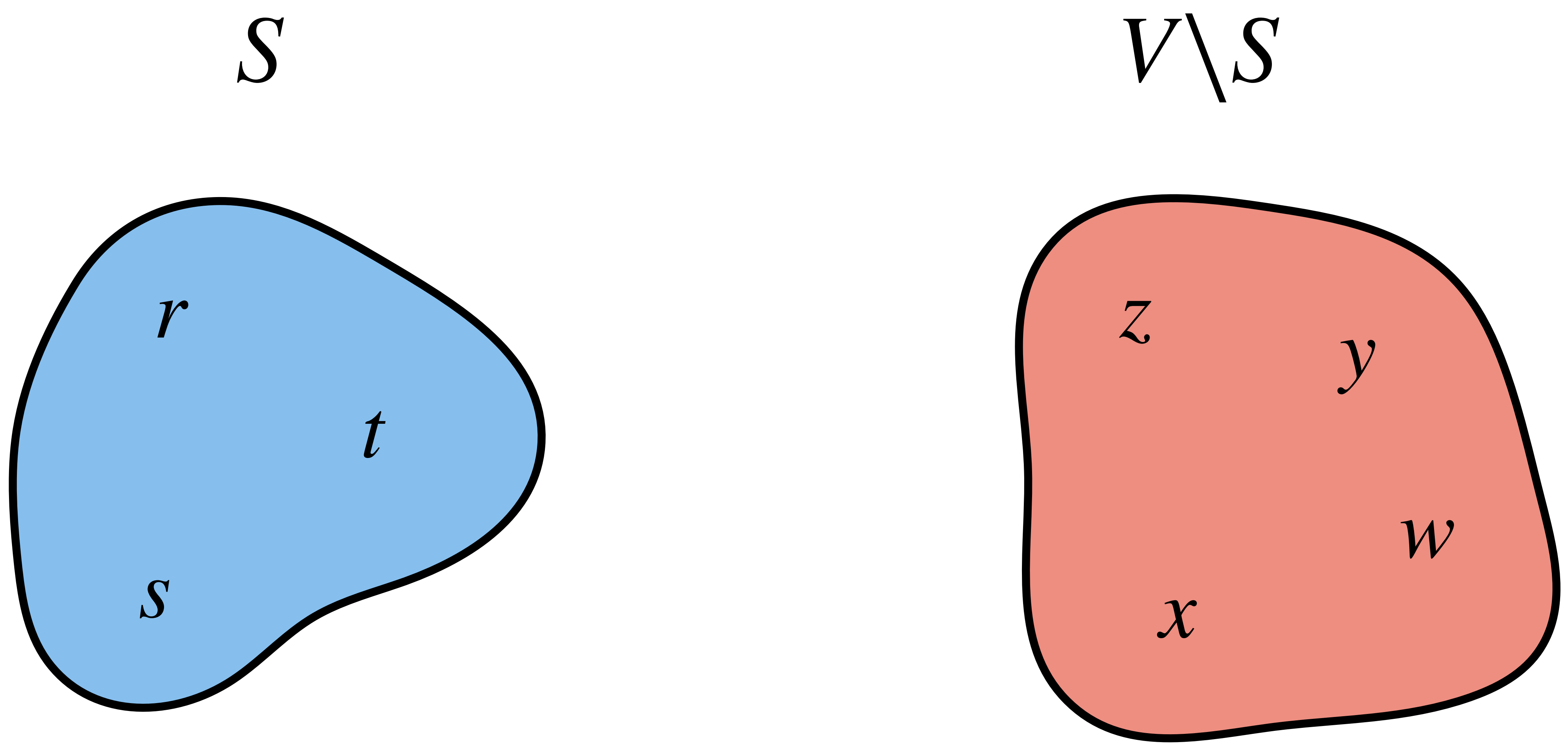
$$\pi[v] = \min_{(u,v) \in E,\ u \in S} d[u] + w(u, v)$$

Add v to S and set $d[v] = \Pi[v]$.

Step 3: Go to **Step 2** if it can be performed.

Dijkstra's Algorithm: Optimization

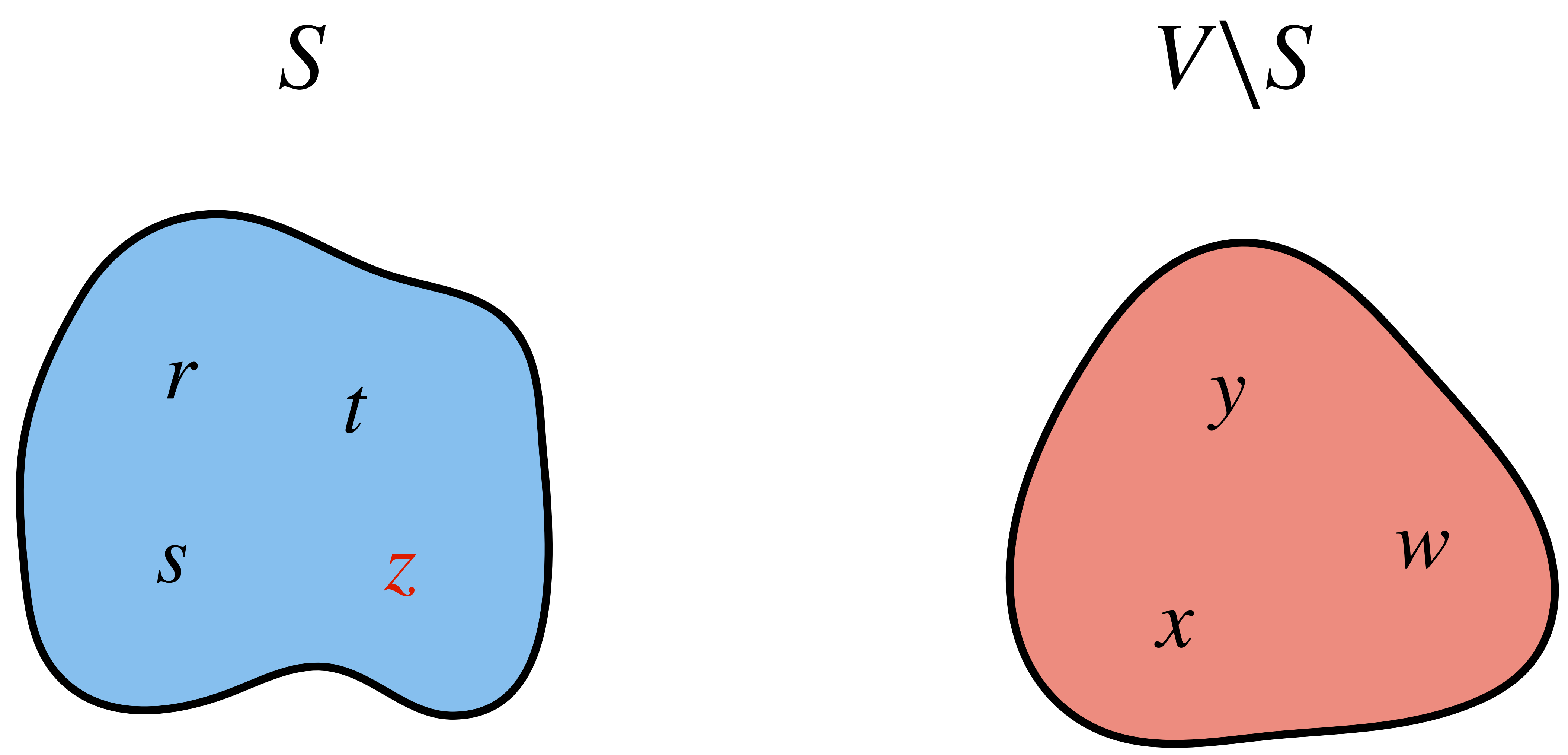
Computed $\pi[v]$ values of $V \backslash S$.



$d[s] = 0, d[r] = 3, d[t] = 2$

$\pi[z] = 4, \pi[y] = 8, \pi[x] = 6, \pi[w] =$ Invalid

Dijkstra's Algorithm: Optimization



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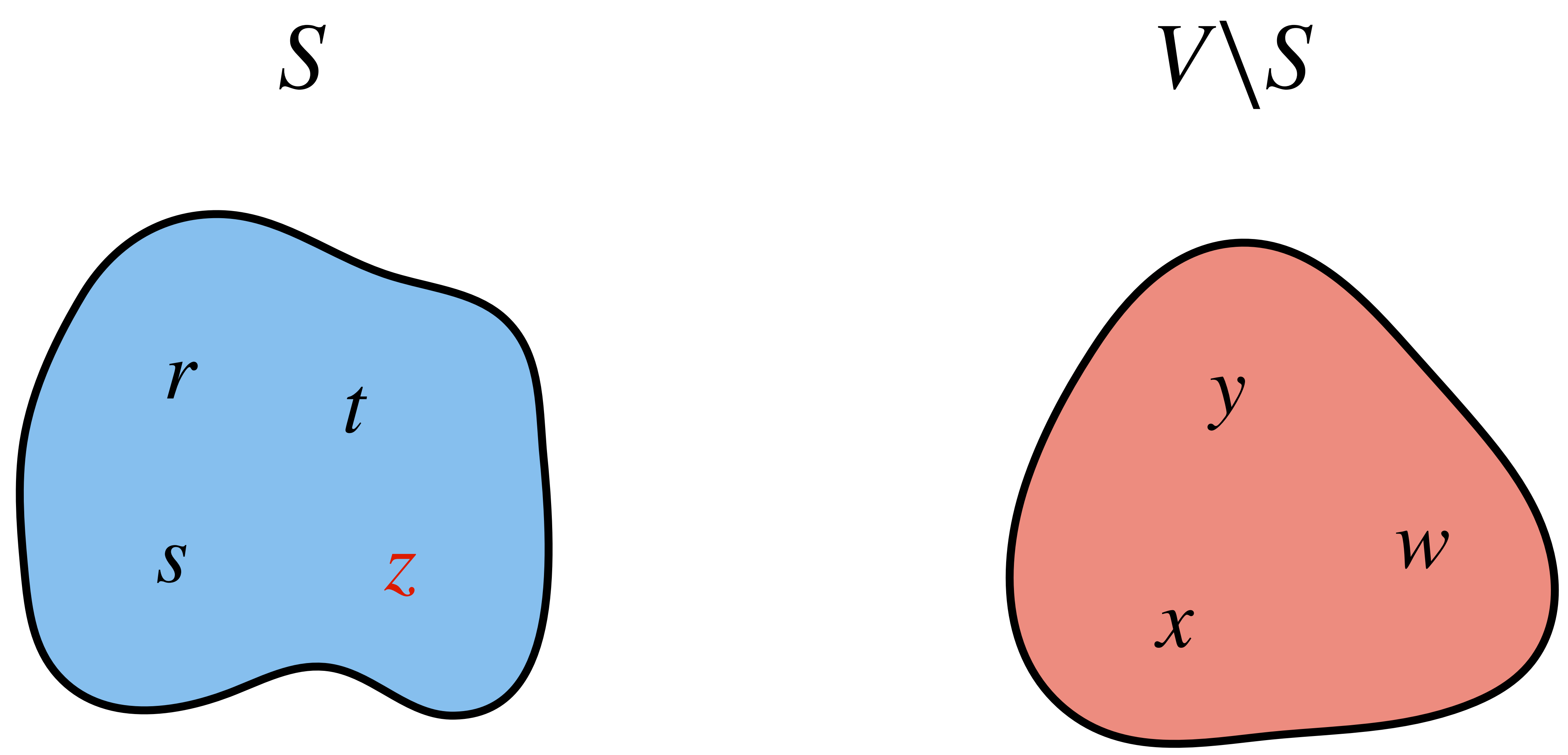
Dijkstra's Algorithm: Optimization

Computing new $\pi[v]$ values after updating S.



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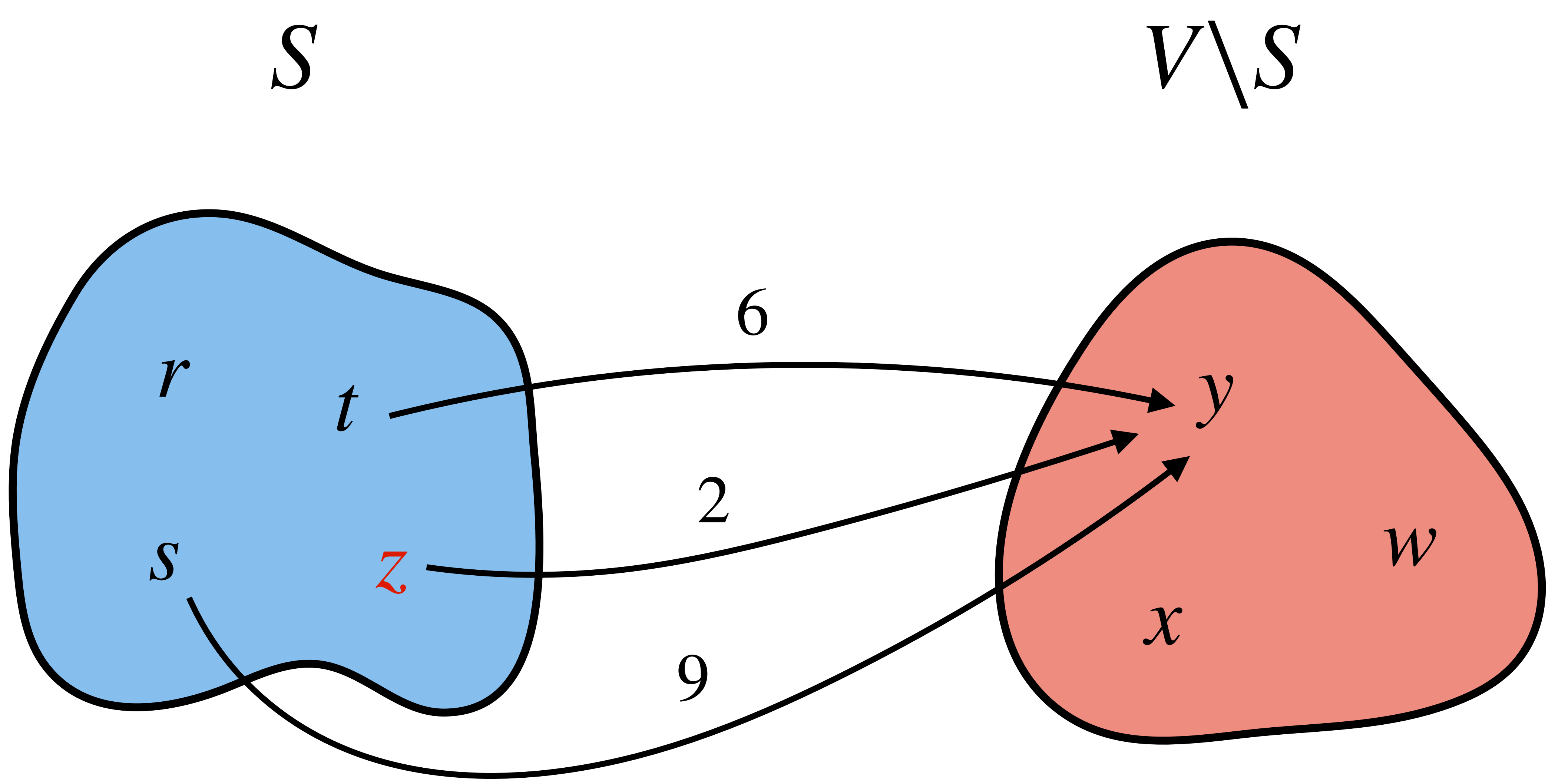

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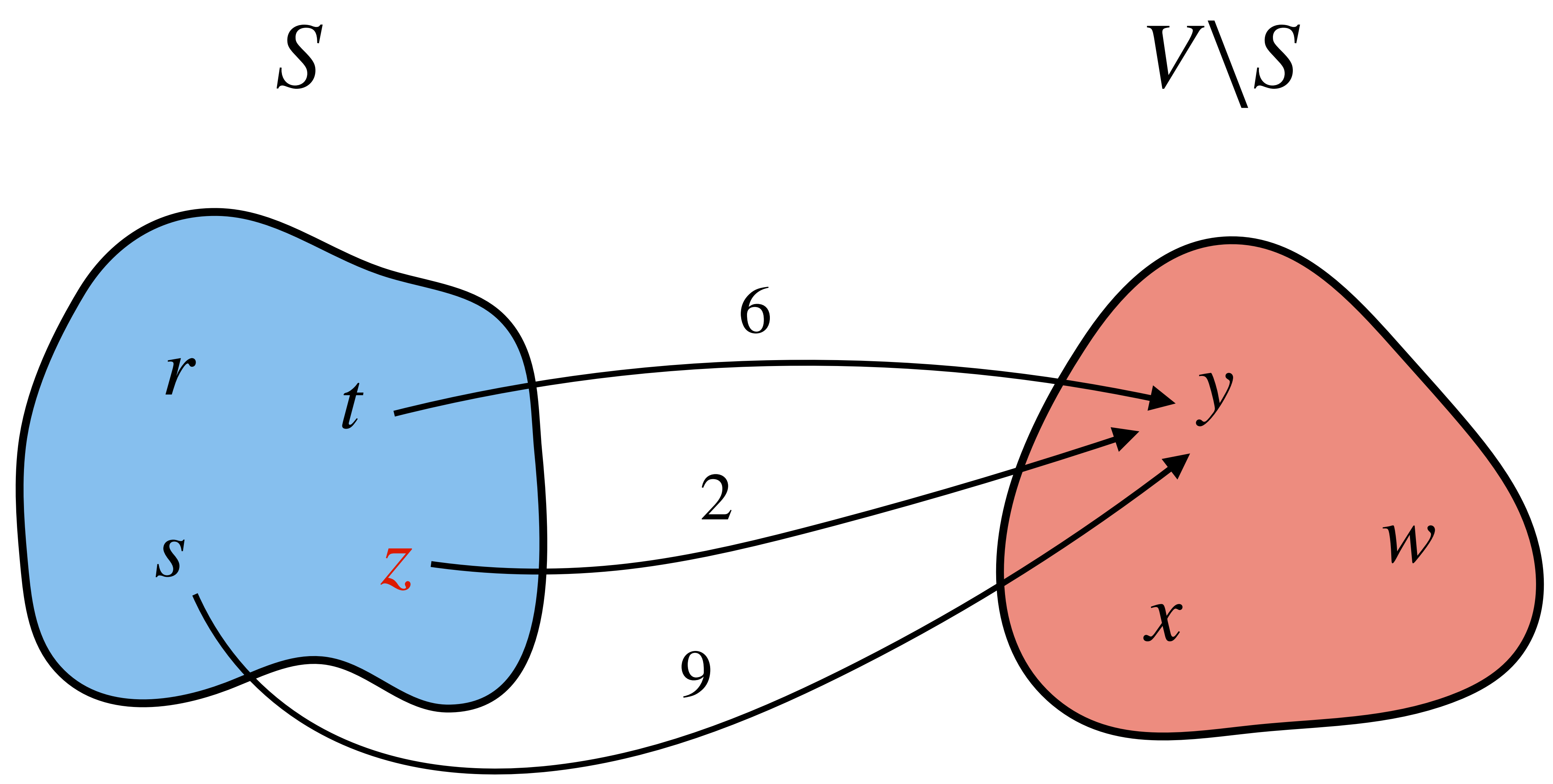

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$\pi[y]$ will be minimum of these values.

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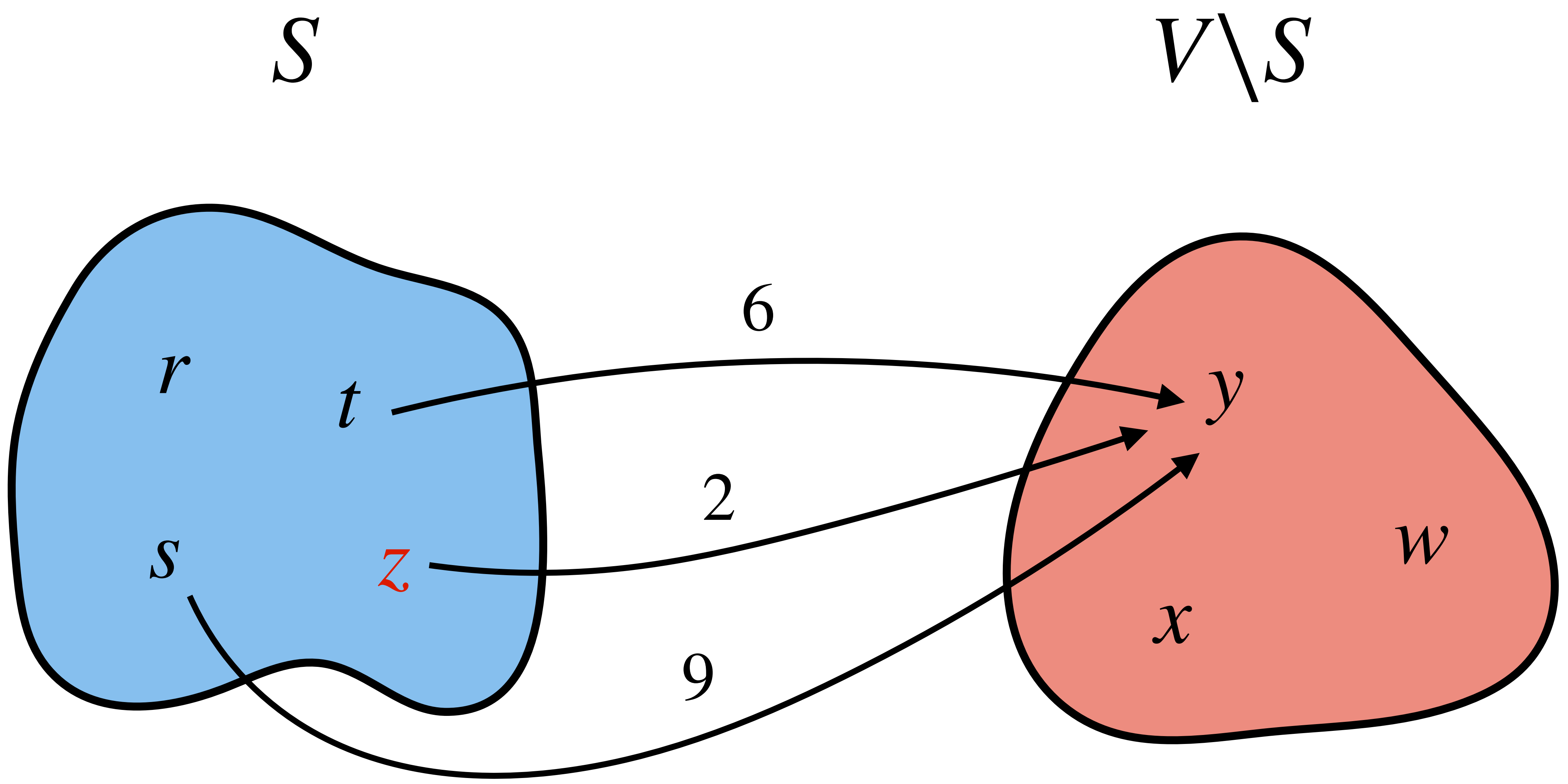

$$d[t] + w(t, y) = 8$$
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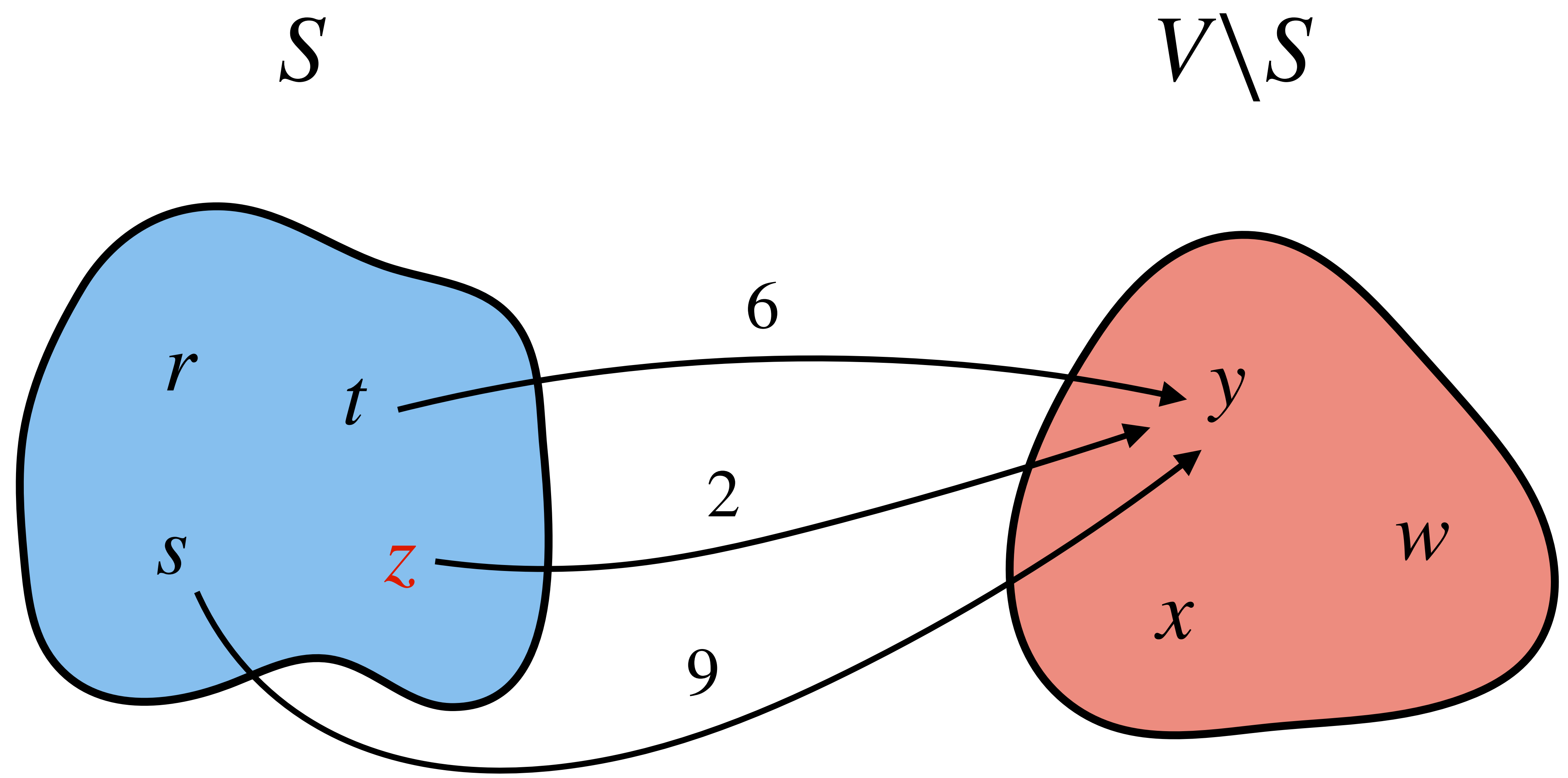

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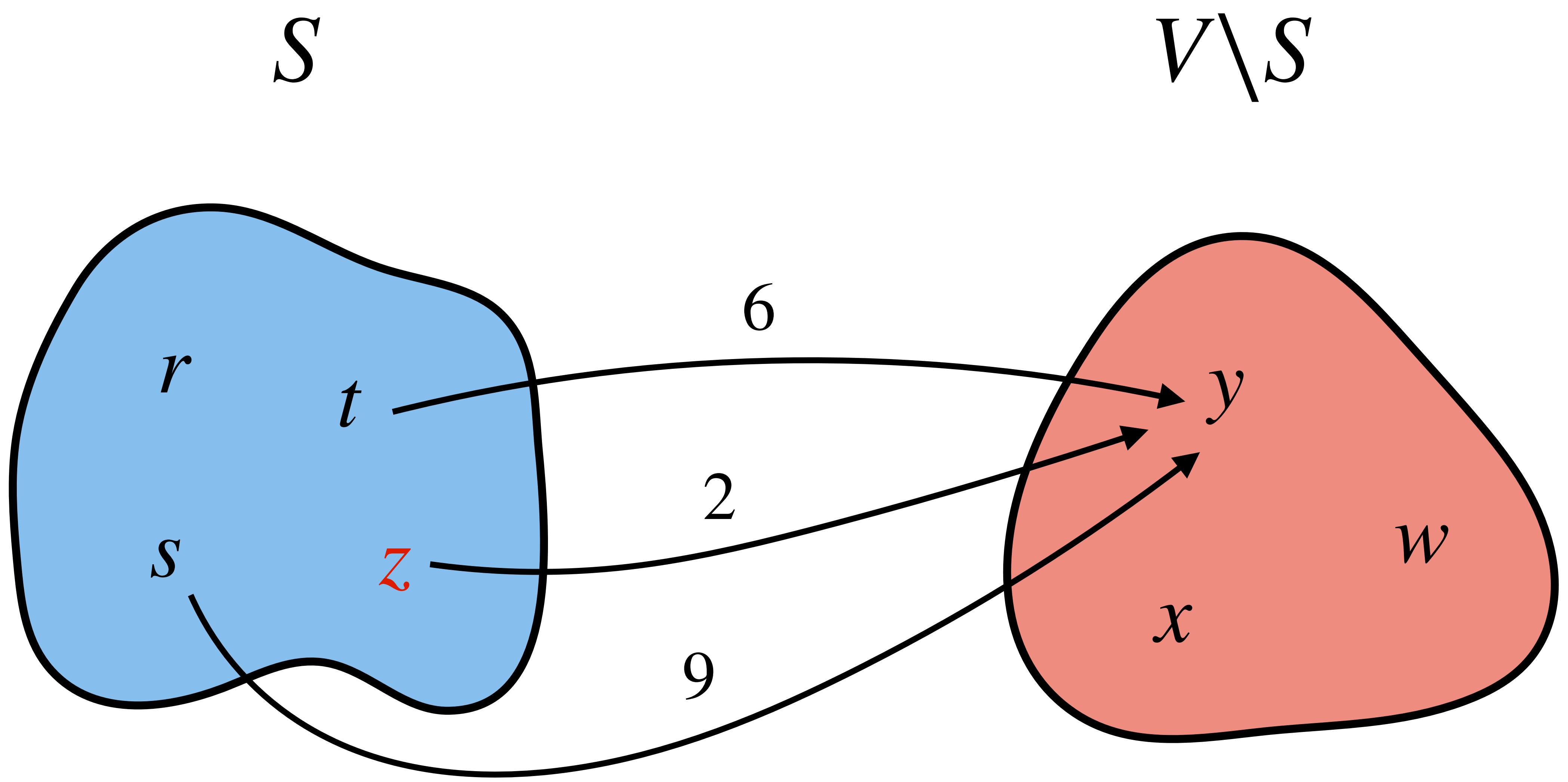

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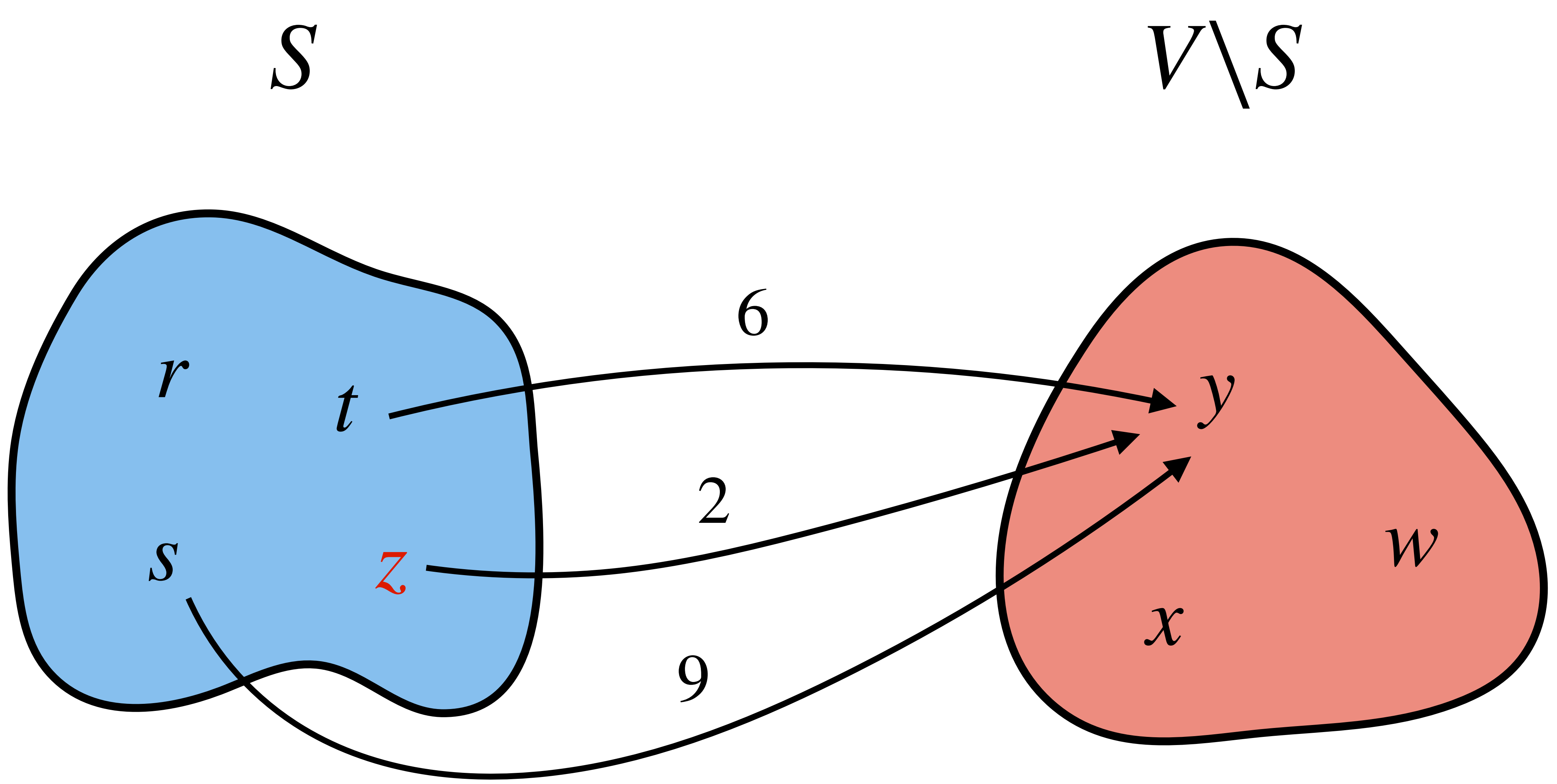

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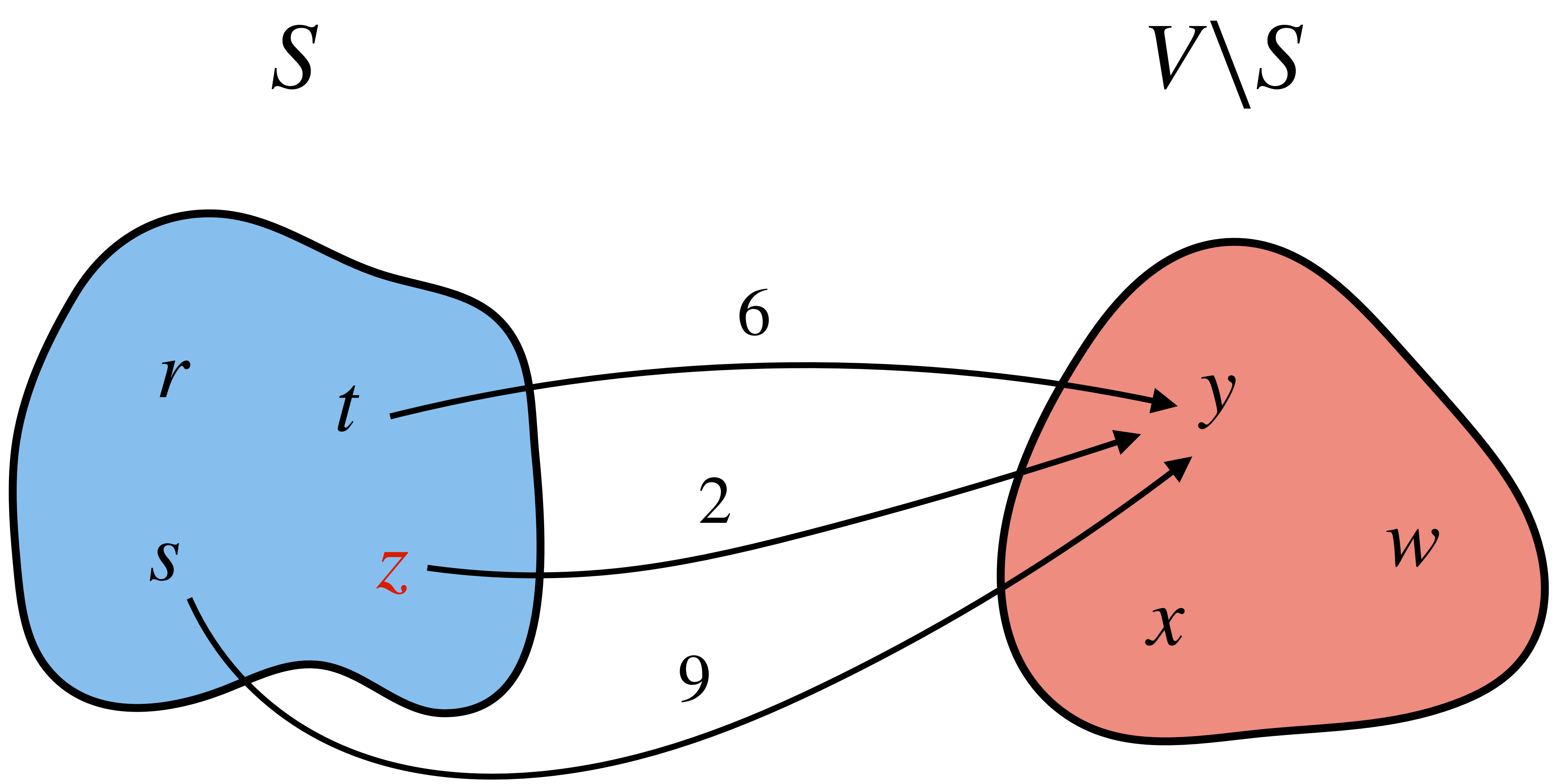

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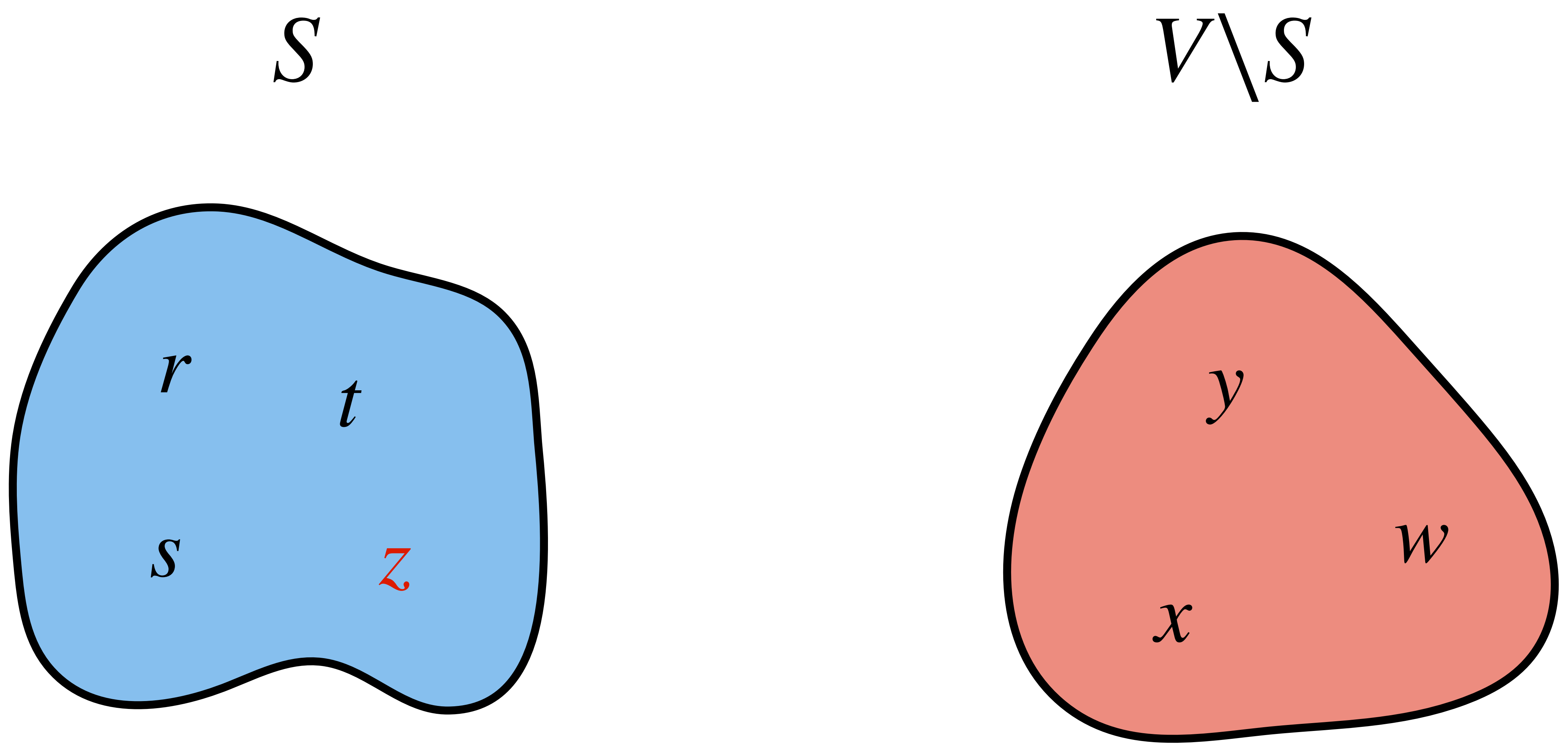

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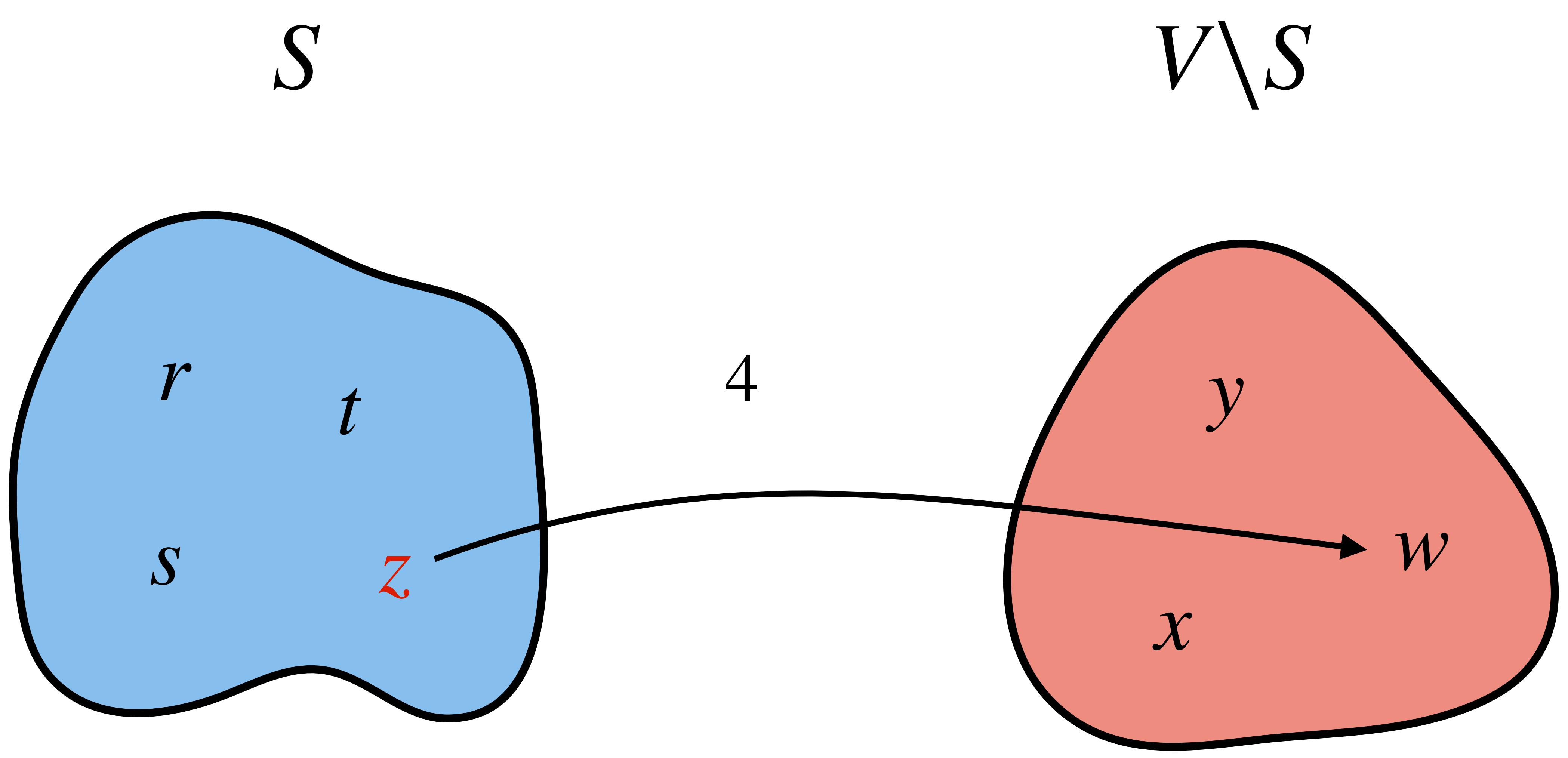

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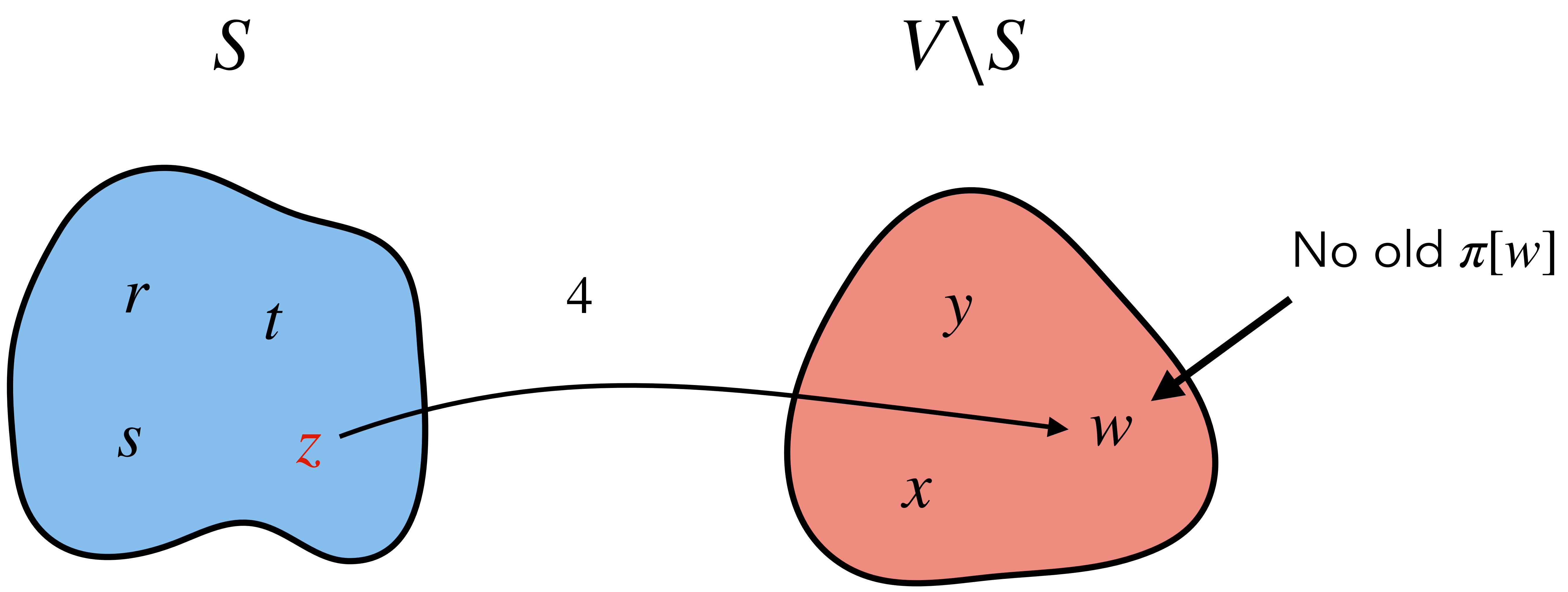

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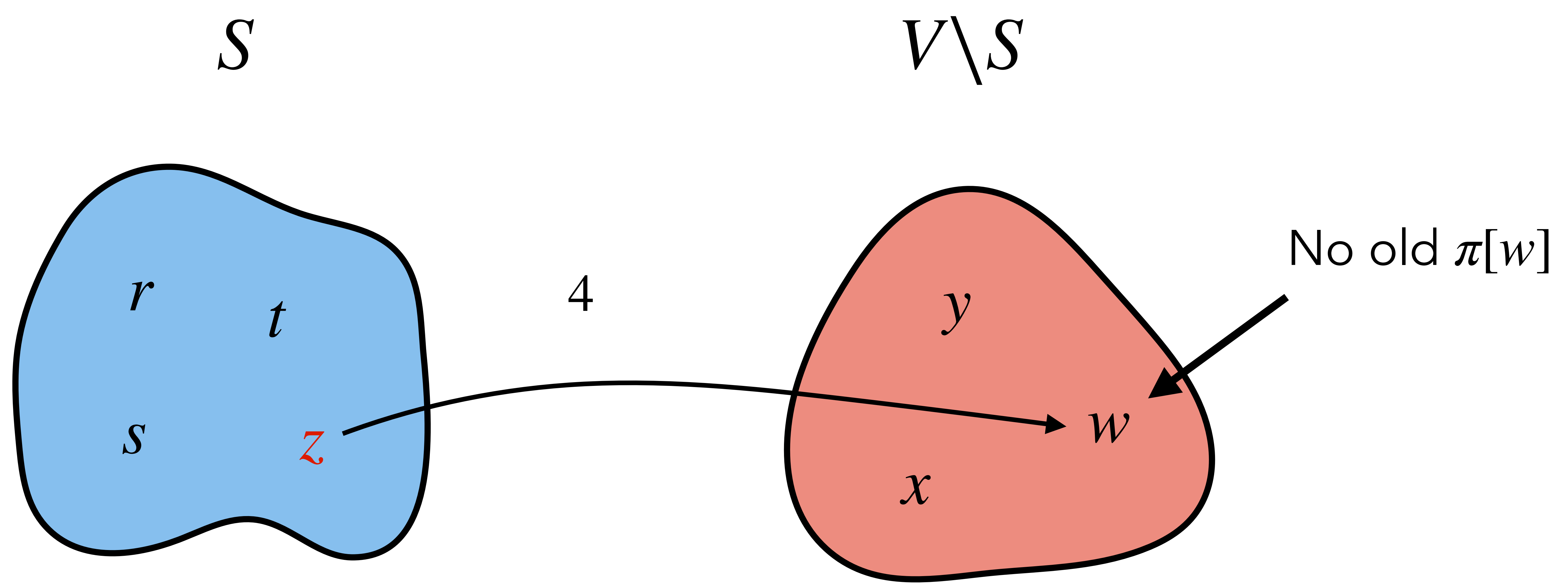

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Idea: Start with $\pi[v] = \infty$ for every $v \in V \backslash \{s\}$ and keep updating π values using above relation.